

1. **(8 points)** Answer the questions below.

(a) **(4 points)** Express $\int_2^5 3^x dx$ as a Riemann sum.

(b) **(4 points)** Evaluate $\int_2^5 3^x dx$.

2. **(8 points)** Evaluate the following limits; if they cannot be evaluated, show why not and describe their behavior.

(a) **(2 points)** $\lim_{x \rightarrow 1} \frac{x^2 - 4x + 3}{\ln x}$

(b) **(3 points)** $\lim_{x \rightarrow \infty} \frac{x^2}{e^x}$.

(c) **(3 points)** $\lim_{x \rightarrow 0} \frac{\cot x}{\ln x}$.

3. **(8 points)** Answer the two following approximation questions.

(a) **(4 points)** Using a well-chosen tangent line approximation, approximate $\sqrt{405}$.

(b) **(4 points)** Construct a function $f(x)$ with $\sqrt[3]{7}$ as a root, and, given the initial point $x_1 = 2$, use Newton's method to determine an approximation x_2 to $\sqrt[3]{7}$.

4. **(8 points)** A conical tank 2 meters in height and one meter in radius is filled to the brim with water and can drain through the bottom at a rate of 0.05 cubic meters per minute.

(a) **(5 points)** How fast is the water level descending?

(b) **(3 points)** After the water has drained to a depth of 1 meter, how fast is the water level descending?

5. (8 points) Answer the following questions related to the shape of the graph of $f(x) = \frac{x^2+2x+6}{x-1}$.

- (3 points) Where is it increasing? Where is it decreasing?
- (3 points) Where is it concave up? Where is it concave down? Does it have any points of inflection?
- (2 points) What are its critical points, and is each a maximum, a minimum, or neither?

6. (8 points) You are constructing a window with the shape shown here of a rectangle of height h surmounted by a semicircle of radius r . You must construct the exterior edge of the window, as well as the three internal struts shown on the figure, using only 21 feet of edging. Find the value of r maximizing the area of the window.

