

1. (12 points) Evaluate the following integrals:

(a) (4 points) $\int_1^2 \frac{1+3x+5x^2}{\sqrt{x}} dx$

(b) (4 points) $\int \frac{\sin x}{\cos^4 x} dx$

(c) (4 points) $\int_e^{e^2} \frac{3}{x(\ln x)^2} dx$

2. (12 points) Consider the function $g(x) = \frac{3x^2+x-2}{x^2-x-6}$.

(a) (5 points) Identify zeroes, vertical asymptotes, and long-term behavior on both sides of this function. Clearly label which is which.

(b) (5 points) Identify the critical points of this function, and indicate whether each is a local maximum, local minimum, or neither.

(c) (2 points) Which if any of the critical points identified above are global maxima or global minima? Show work or explain.

3. **(12 points)** Compute the following derivatives:

(a) **(4 points)** Given $f(t) = (\arcsin t)e^{\tan(t)}$, find $f'(t)$.

(b) **(4 points)** Find $\frac{d}{dx} \int_2^x s^3(\cos s)ds$.

(c) **(4 points)** Find $\frac{d}{d\theta} \sqrt{\ln \sec \theta}$.

4. **(12 points)** Alice is 10 miles north of the university walking south at 4 miles per hour. Bob is currently at the university bicycling east at 12 miles per hour.

(a) **(9 points)** Are Alice and Bob getting closer to or further from each other, and at what rate are they doing so?

(b) **(9 points)** In half an hour, will Alice and Bob be getting closer to or further from each other, and at what rate will they doing so?

5. **(12 points)** Answer the following questions about the function $h(x) = e^{\sqrt{x-4}} + 2$.
- (a) **(4 points)** What is the domain of $h(x)$?

 - (b) **(4 points)** What is the range of $h(x)$?

 - (c) **(4 points)** Where is $h(x)$ differentiable?
6. **(12 points)** A casserole is heated to 160°F and then removed from the oven into a 70°F room. After half an hour, it has cooled to 100°F .
- (a) **(4 points)** Construct a function modeling the temperature of the casserole t hours after it is removed from the oven.

 - (b) **(4 points)** How quickly is the casserole cooling one hour after it is removed from the oven?

 - (c) **(4 points)** How long will it take the casserole to cool to 80°F ?

7. **(12 points)** The equation $x^2 + 4xy + 9y^2 - 2x + 5y = 179$. describes an ellipse.

(a) **(9 points)** Find a formula on the ellipse for $\frac{dy}{dx}$ in terms of x and y .

(b) **(3 points)** Find the equation of the tangent line to this ellipse at the point $(1, 4)$.

8. **(12 points)** You are asked to construct an box with a square base and an open top, containing 16 cubic feet. If material for the bottom of the box costs \$2 per square foot, and material for the sides costs \$1 per square foot, what dimensions for the box will minimize the cost?

9. **(12 points)** Determine the following limits.

(a) **(4 points)** *Using the difference quotient*, find the derivative with respect to x of $f(x) = 2x^2 - 3x + 2$. You may not use L'Hôpital's rule for this problem.

(b) **(4 points)** Evaluate $\lim_{x \rightarrow 1} \frac{\ln x}{x^2 - 1}$ or demonstrate that it cannot be evaluated.

(c) **(4 points)** Evaluate $\lim_{x \rightarrow -\infty} \frac{x^2 - 3x + 2}{4x - 2}$ or demonstrate that it cannot be evaluated.

10. **(12 points)** Let $f(x) = (\ln |x|)^2$.

(a) **(5 points)** Where is $f(x)$ increasing? Where is it decreasing?

(b) **(7 points)** Determine $f(x)$'s concavity and identify points of inflection.