

This test is closed-book and closed-notes. No calculator is allowed for this test. When answering the questions you are only allowed to use tools and techniques that we have discussed up to this point, i.e. no advanced differentiation techniques. For full credit show all of your work (legibly!), unless otherwise specified.

For the purposes of this exam, “familiar functions” includes all arithmetic operations as well as trigonometric functions, inverse trigonometric functions and natural logarithms.

The problems are in no particular order, and it is suggested that you look at all of them before beginning to answer any.

1. (8 points) Let $f(x) = \begin{cases} ax & \text{if } x \leq 3 \\ 2^x & \text{if } 3 < x \leq 4. \\ x^2 + b & \text{if } x > 4 \end{cases}$.

What choices of a and b will make this function continuous?

1	
2	
3	
4	
5	
6	
7	
8	
Σ	

2. **(8 points)** Answer the following questions for the function $f(t) = 4 \sin(2\pi t) + 1$.

(a) **(3 points)** What are its domain and range?

(b) **(2 points)** Is it odd, even, both, or neither? Briefly justify your answer.

(c) **(3 points)** What are its amplitude and period?

3. **(8 points)** Let $f(x) = 2x^2 - 3x$.

(a) **(6 points)** *Using the difference quotient*, find $f'(x)$.

(b) **(2 points)** Find the equation of the tangent line to $f(x)$ at the point $(-1, 5)$.

4. **(8 points)** Evaluate the following limits; when a limit can not be evaluated, explain why or describe its behavior.

(a) **(2 points)** $\lim_{u \rightarrow 1} \frac{(u^2 - 4u + 3)}{(u - 1)}$

(b) **(2 points)** $\lim_{x \rightarrow +\infty} \frac{5x^3 - 2x + 1}{4x^3 + 2x^2}$

(c) **(2 points)** $\lim_{\theta \rightarrow \frac{\pi}{2}^+} 3 \tan \theta$

(d) **(2 points)** $\lim_{t \rightarrow 2} \frac{t^2 - 4}{t + 3}$

5. **(8 points)** Let $f(t) = \frac{-3t^2+12}{t+2}$.

(a) **(1 point)** Find $\lim_{t \rightarrow -2} f(t)$.

(b) **(4 points)** Using epsilon-delta methods, justify your result above.

(c) **(3 points)** State the mathematical definition of the statement $\lim_{x \rightarrow -\infty} f(x) = +\infty$.

6. **(8 points)** Given the function $g(x) = \frac{(x-5)(x-2)}{(x^2+3)(x-2)(x+4)}$, answer the following questions preparatory to sketching the functions.

(a) **(2 points)** What is the domain of the function?

(b) **(2 points)** What are all the zeroes of the function?

(c) **(2 points)** What are all the vertical asymptotes of the function?

(d) **(2 points)** Describe, either in words or symbolically, the long-term behavior of the function in each direction.

7. **(8 points)** Transylvania Polygnostic University currently has 3000 students. Enrollment is expected to rise by 4% each year.

(a) **(3 points)** Create a function $f(t)$ to describe the expected number of students t years from now.

(b) **(5 points)** How many years will it take for enrollment to reach 4000 students?

8. **(4 point bonus)** Using the difference quotient, find the derivative of the function $f(x) = \sqrt[3]{x}$.