This test is closed-book and closed-notes. No calculator is allowed for this test. For full credit show all of your work (legibly!), unless otherwise specified.

For the purposes of this exam, all answers must be in terms of "familiar functions", which includes all arithmetic operations as well as trigonometric functions, inverse trigonometric functions, and natural logarithms. Algebraic simplification of answers is generally unnecessary.

The problems are in no particular order, and it is suggested that you look at all of them before beginning to answer any.

1. **(8 points)** Find an equation of the tangent line to the curve $y = \sqrt{25 - x^2}$ at $(3, 4)$.

2. **(8 points)** The height in meters of a vertically-moving balloon at time t, measured in seconds, is given by the formula $s(t) = t^3 + t^2 - 8t + 4$.

   (a) **(2 points)** How quickly and in what direction is the balloon moving after 1 second?

   (b) **(2 points)** What is the balloon's acceleration after 3 seconds?

   (c) **(4 points)** At what times is the balloon moving downwards?

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3. **(8 points)** Imre is twelve miles north of the Parliament, jogging southwards at six miles per hour; János is five miles to the east of Parliament, walking eastwards at three miles per hour.

   (a) **(4 points)** Is the distance between Imre and János increasing or decreasing, and at what rate?

   (b) **(4 points)** In an hour, will the distance between Imre and János increasing or decreasing, and at what rate?

4. **(8 points)** Differentiate $\frac{\arctan t}{\ln(\sin t)}$ with respect to t.

5. **(8 points)** The *folium of Descartes* is a curve satisfying the equation $x^3 + y^3 - 5xy = 0$.

   (a) **(6 points)** Find a formula for $\frac{dy}{dx}$ on this curve.

   (b) **(2 points)** Identify conditions on x and y for the tangent lines to the folium to be horizontal and vertical (label which is which!).

6. **(8 points)** A collection of biological samples is taken from a $-200°$F deep-freeze into a $50°$F lab. After 10 minutes it has warmed up to -150°F.

   (a) **(4 points)** Produce a function $T(t)$ modeling the samples' temperature t minutes after they are brought into the lab.

   (b) **(2 points)** The samples will become biologically active when they reach $0°$F. How long will it take for this to occur?

   (c) **(2 points)** How quickly are the samples' temperature changing ten minutes after being brought into the lab?

(d) **(8 points)** Calculate $\frac{d}{dx}\arcsin(x^2 \tan x)$.

(e) **(4 point bonus)** The power rule is traditionally only proven for positive integers. Using other rules, however, one builds the more general form of the power rule. In the problems below, assume the only derivative you can take directly is $\frac{d}{dx}x^n$ when n is a positive integer.

i. **(2 points)** Find $\frac{d}{dx}x^n$ when n is a negative integer.

ii. **(2 points)** Find $\frac{d}{dx}x^n$ when n is a fraction of the form $\frac{1}{k}$, for positive integer k.