

This test is closed-book and closed-notes. No calculator is allowed for this test. For full credit show all of your work (legibly!), unless otherwise specified.

For the purposes of this exam, all answers must be in terms of “familiar functions”, which includes all arithmetic operations as well as trigonometric functions, inverse trigonometric functions, and natural logarithms. Algebraic and trigonometric simplification of answers is generally unnecessary.

The problems are in no particular order, and it is suggested that you look at all of them before beginning to answer any.

1. **(8 points)** The horizontal displacement in meters of a particle at time t , measured in seconds, is given by the formula $s(t) = t^3 - 8t^2 + 16t$.
 - (a) **(2 points)** What is the particle’s velocity after 2 seconds?

 - (b) **(4 points)** At what times is the particle moving to the right?

 - (c) **(2 points)** What is the particle’s acceleration after 5 seconds?

2. **(8 points)** A bar of iron is taken from a 1400°F furnace into a 100°F metalworking studio. After 5 minutes it has cooled to 800°F .
 - (a) **(4 points)** Produce a function $T(t)$ modeling the bar’s temperature t minutes after removal from the furnace.

 - (b) **(2 points)** How quickly is the bar’s temperature changing immediately upon removal from the furnace?

 - (c) **(2 points)** The metal can be worked as long as it is hotter than 1000°F . How soon after the bar is removed from the furnace does it become unworkable?

3. **(8 points)** Calculate $\frac{d}{dx}(\sqrt{x} \csc(\ln x))$.

4. **(8 points)** The *conchoid of de Sluze* is a curve satisfying the equation $(x - 1)(x^2 + y^2) = 4x^2$.

(a) **(6 points)** Find a formula for $\frac{dy}{dx}$ on this curve.

(b) **(2 points)** Identify conditions on x and y for the tangent lines to the conchoid to be horizontal and vertical (label which is which!).

5. **(8 points)** Find an equation of the tangent line to the curve $y = 3 \sin(\pi x)$ at $(\frac{1}{6}, \frac{3}{2})$.

6. **(8 points)** A thief is sneaking eastwards along a wall at two miles per hour. However, 3 miles east and 4 miles south of him, a guard with a telescope is watching.
- (a) **(4 points)** The guard needs to swivel the telescope to keep it trained on the thief. How quickly should she be turning it?

(b) **(4 points)** How quickly is the distance between the thief and guard changing?

7. **(8 points)** Differentiate $\frac{(\arcsin u) \sqrt[5]{u}}{\tan u}$ with respect to u .