

This test is closed-book and closed-notes. No calculator is allowed for this test. For full credit show all of your work (legibly!), unless otherwise specified.

For the purposes of this exam, all answers must be in terms of “familiar functions”, which includes all arithmetic operations as well as trigonometric functions, inverse trigonometric functions, and natural logarithms. Algebraic and trigonometric simplification of answers is generally unnecessary.

The problems are in no particular order, and it is suggested that you look at all of them before beginning to answer any.

1. **(12 points)** You are constructing an open-topped cardboard box with a square base which must have a volume of thirty-two cubic feet. What is the least amount of cardboard you could use to do so?

2. (10 points) Evaluate the following limits; if they cannot be evaluated, show why not.

(a) (3 points) $\lim_{x \rightarrow 3} \frac{x^2 - 2x + 1}{2^x - 1}$

(b) (4 points) $\lim_{x \rightarrow 0^-} x \cot x$

(c) (3 points) $\lim_{x \rightarrow 0} \frac{x^3 + x^2}{e^x - x - 1}$

3. (10 points) Find approximations to the following values using appropriate linearizations.

(a) (5 points) Find a rational approximation to $\ln(e^3 + 0.1)$.

(b) (5 points) Find a rational number approximately equal to $\sqrt{98}$.

4. **(10 points)** Answer the following questions related to the shape of the graph of $f(x) = x^3 + 6x^2 - 15x + 7$.

(a) **(3 points)** Where is it increasing? Where is it decreasing?

(b) **(3 points)** What are its critical points, and is each a local maximum, a local minimum, or neither?

(c) **(4 points)** Where is it concave up? Where is it concave down? Does it have any points of inflection?

5. **(10 points)** Answer the following questions about approximation:

(a) **(5 points)** Starting with an initial value of 2, use two iterations of Newton's method to approximate a zero of $f(x) = x^3 - x + 4$. Your answer need not be arithmetically simplified.

(b) **(5 points)** Choose an integer value of x_0 approximating $\sqrt{17}$ as well as possible. Use one step of Newton's method to develop a better rational approximation x_1 .

6. **(10 points)** Answer the following questions:

(a) **(5 points)** Find an antiderivative of the function $f(x) = 2x^4 + \cos x - \frac{1}{x^2+1}$.

(b) **(5 points)** Determine a region whose area is $\lim_{n \rightarrow \infty} \sum_{i=1}^n \left(\frac{5}{n}\right) e^{4+\frac{5i}{n}}$.