1. **(4 points)** *Identify the domain of the following functions:*

   (a) **(2 points)** $g(t) = \sqrt{3t-2}$.

   The obvious impediment to the existence of this function is when $3t-2$ is negtive, since square roots of negative numbers are specifically not doable in the real numbers. So this function exists only when $3t-2 \geq 0$. Algebraic manipulation of this expression gives $3t \geq 2$, to $t \geq \frac{2}{3}$; this can be alternatively expressed as the interval notation $[\frac{2}{3}, \infty)$.

   (b) **(2 points)** $f(x) = \frac{4x}{2^x-1}$.

   Here, the only clear impediment to the existence of this function is the division: we must be careful to make sure the denominator is nonzero. Thus, we require that $2^x - 1 \neq 0$, which algebraically simplifies to $2^x \neq 1$ or $x \neq \log_2 1$ This is a known logarithm: the logarithm in any base of 1 is 0 (since $a^0 = 1$ for all positive a), so this condition becomes $x \neq 0$, or, in interval notation, $(-\infty, 0) \cup (0, \infty)$.

2. **(4 points)** *Sunspot intensity fluctuates over an 11-year cycle. The average radiance of the sun is* 1366 *watts per square meter, and the sunspot cycle causes variance around this average of* ± 0.75 *watts per square meter. Create a function $f(t)$ which would serve to model the sun's radiance over time, with t being measured in years (you need not worry about phase shift).*

   The sunusoidal function described here has an amplitude of 0.75, a vertical shift of 1366, and a period of 11. We thous take a standard sine function (amplitude 1, period 2π, vertical shift 0), and enact upon it a vertical-stretch of 0.75, a vertrical shift of 1366, and a horizontal stretch of $\frac{11}{2\pi}$ to get the desiired properties. The result of these actions is the function $f(t) = 0.75 \sin\left(\frac{2\pi}{11}t\right) + 1366$.

3. **(4 points)** *If $f(x) = x^2 - x$, evaluate and simplify the expression $\frac{f(2+h)-f(2)}{h}$.*

$$\begin{aligned} \frac{f(2+h)-f(2)}{h} &= \frac{[(2+h)^2 - (2+h)] - [2^2 - 2]}{h} \\ &= \frac{[4 + 4h + h^2 - (2+h)] - 2}{h} \\ &= \frac{3h + h^2}{h} = 3 + h \end{aligned}$$

4. **(8 points)** *Variola Modo is breeding a new species of bacterium whose population doubles every 10 minutes. He starts with a sample of 40 bacteria.*

   (a) **(4 points)** *Construct a function $f(t)$ to describe the bacteria's population after t minutes.*

   We want $f(0)$ to be 40, and $f(10)$ to be $40 \cdot 2^1$, and $f(20)$ to be $40 \cdot 2^2$, and so forth; since this is a population growth function, an exponential mdoel is appropriate, and we see from these examples (or familiarity with doubling-time formulae) that an appropriate exponential model is $f(t) = 40 \cdot 2^{\frac{t}{10}}$.

(b) **(4 points)** *Use your function to determine the number of minutes it will take for the population to reach* 800 *bacteria. Your answer should be in the simplest calculatable form.*

We want to solve for the value of t when $f(t) = 800$:

$$\begin{aligned} 40 \cdot 2^{\frac{t}{10}} &= 800 \\ 2^{\frac{t}{10}} &= 20 \\ \frac{t}{10} &= \log_2 20 \\ t &= 10 \log_2 20 = 10 \frac{\ln 20}{\ln 2} \approx 43.2 \text{ minutes} \end{aligned}$$