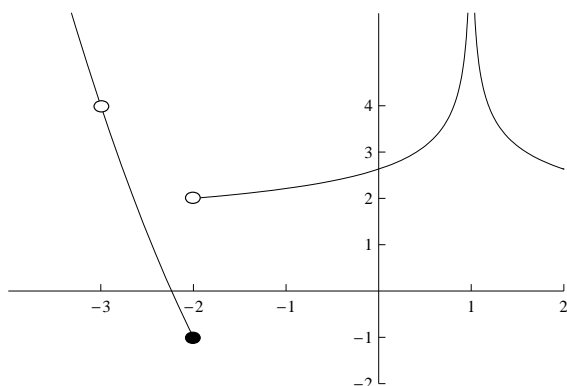


1. (4 points) For the plot of $g(x)$ shown below, indicate whether or not each of the following quantities can be evaluated. If they can be evaluated, compute their values. If they cannot be evaluated, explain why.



$g(-2) = -1$, as indicated by the solid dot.

$\lim_{x \rightarrow -3^-} g(x) = 4$, since slightly to the left of the open circle at $(-3, 4)$, $f(x)$ is close to 4.

$\lim_{x \rightarrow -2} g(x)$ does not exist because of a jump discontinuity.

$\lim_{x \rightarrow 1^+} g(x)$ does not exist, since $g(x)$ increases without bound as x approaches 1 from above.

2. (4 points) The table below describes the distance traveled on a cross-country drive over a six-hour period.

Time	Noon	1PM	2PM	3PM	4PM	5PM	6PM
Distance in miles	0	65	150	230	270	350	420

Calculate these properties of the trip:

- (a) (2 points) The average speed over the course of the entire six-hour period.
In six hours, the car has traveled 420 miles, so the average speed is $\frac{420}{6} = 70$ miles per hour.
- (b) (2 points) The average speed between 2:00 PM and 4:00 PM.
In two hours, the car has traveled $270 - 150 = 120$ miles, so the average speed during this period is $\frac{120}{2} = 60$ miles per hour. This can also be expressed as the more standard average-speed formulation $\frac{270-150}{4-2} = \frac{120}{2} = 60$.
3. (4 points) Evaluate the following limits, or explicitly state that they do not exist and explain why they cannot be evaluated.

(a) (2 points) $\lim_{x \rightarrow 1} \frac{x^2 - 4}{x^3 - 3x + 4}$.

1 is in the domain of this rational function, so we simply evaluate it at 1 to get $\frac{1^2 - 4}{1^3 - 3 \cdot 1 + 4} = \frac{-3}{2}$.

(b) **(2 points)** $\lim_{s \rightarrow 4} \sqrt{4 - s}$.

For s slightly larger than 4, $4 - s$ is negative so $\sqrt{4 - s}$ is not evaluatable in the real numbers. Thus, this limit does not exist since the right-side limit, in particular, does not exist.

4. **(4 points)** Using epsilon-delta methods, prove that $\lim_{x \rightarrow 2} \frac{2x^2 - 7x + 6}{x - 2} = 1$.

Given an ϵ , we want to derive a δ such that if x is within δ of 2, then $\frac{2x^2 - 7x + 6}{x - 2}$ is within ϵ of 1. We start with the latter restriction, and then try to derive, via reversible steps, the necessary value of x :

$$\begin{aligned} \left| \frac{2x^2 - 7x + 6}{x - 2} - 1 \right| &< \epsilon \\ \left| \frac{(2x - 3)(x - 2)}{x - 2} - 1 \right| &< \epsilon \\ |2x - 3 - 1| &< \epsilon \text{ and } x \neq 2 \\ |2x - 4| &< \epsilon \text{ and } x \neq 2 \\ |x - 2| &< \frac{\epsilon}{2} \text{ and } x \neq 2 \\ 0 &< |x - 2| < \frac{\epsilon}{2} \end{aligned}$$

so an ϵ -bound on $\frac{2x^2 - 7x + 6}{x - 2}$ is equivalent to an $\frac{\epsilon}{2}$ -bound on x , so any choice of $\delta \leq \frac{\epsilon}{2}$ will suffice.

5. **(4 points)** Find a value of a such that the given function is continuous everywhere:

$$f(t) = \begin{cases} at & \text{if } t \leq 3 \\ t^2 - a & \text{if } t > 3 \end{cases}$$

Since the only possible discontinuity exists at $t = 3$, to ensure continuity throughout, we must make sure the left- and right-side limits at $t = 3$ are identical. The limit from the left at $t = 3$ is the behavior to the left of $t = 3$ evaluated at 3, that is to say, $a \cdot 3$. The right-side limit, on the other hand, is $3^2 - a$. Thus, $3a = 9 - a$, so $4a = 9$, so $a = \frac{9}{4}$.