1. **(8 points)** Solve the problems given below.

   (a) **(4 points)** Given $f(x) = (x^5 + e^x)(x + 3)$, find $f'(x)$.
   $f'(x) = \frac{d}{dx}\left[(x^5 + e^x)(x + 3)\right]$, which we can evaluate using the product rule:

$$\begin{aligned} f'(x) &= \frac{d}{dx}\left[(x^5 + e^x)(x + 3)\right] \\ &= \left[\frac{d}{dx}(x^5 + e^x)\right](x + 3) + (x^5 + e^x)\frac{d}{dx}(x + 3) \\ &= (5x^4 + e^x)(x + 3) + (x^5 + e^x)(1) \end{aligned}$$

   (b) **(4 points)** *Find* $\frac{d}{dt}\frac{t^2-1}{e^t}$.
   Using the quotient rule:

$$\begin{aligned} \frac{d}{dt}\frac{t^2 - 1}{e^t} &= \frac{(e^t)\left[\frac{d}{dt}(t^2 - 1)\right] - (t^2 - 1)\frac{d}{dt}e^t}{(e^t)^2} \\ &= \frac{(e^t)(2t) - (t^2 - 1)e^t}{(e^t)^2} \\ &= \frac{2t - t^2 + 1}{e^t} \end{aligned}$$

   The last step is not strictly necessary.

2. **(4 points)** *Determine the values of the following limits if possible; if they can not be evaluated, explain why.*

   (a) **(2 points)** $\lim_{x\to-\infty} \frac{-2x^3-3x}{x^2-4}$
   Considering only the highest order terms, we can see that $\lim_{x\to-\infty} \frac{-2x^3-3x}{x^2-4} = \lim_{x\to-\infty} \frac{-2x^3}{x^2} = \lim_{x\to-\infty} -2x$; however, as x decreases without bound, $-x$ increases without bound; thus, the limit does not exist since $\frac{-2x^3-3x}{x^2-4}$ becomes arbitrarily large as x becomes extremely negative; colloquially, we would write $\lim_{x\to-\infty} \frac{-2x^3-3x}{x^2-4} = \infty$, but this is not actually an evaluation of the limit.
   If considering only the highest order terms does not appeal, one can instead divide the numerator and denominator by x^2 and reduce the limit using the known fact that $\lim_{x\to\pm\infty} \frac{1}{x^k} = 0$ for positive k:

$$\lim_{x\to-\infty} \frac{-2x^3 - 3x}{x^2 - 4} = \lim_{x\to-\infty} \frac{-2x - \frac{3}{x}}{1 - \frac{4}{x}} = \lim_{x\to-\infty} \frac{-2x - 0}{1 - 0} = \lim_{x\to-\infty} -2x$$

   (b) **(2 points)** $\lim_{x\to\infty} \frac{5x^4-2x^2+x}{2x^4+1}$
   Considering only the highest order terms, we can see that $\lim_{x\to\infty} \frac{5x^4-2x^2+x}{2x^4+1} = \lim_{x\to\infty} \frac{5x^4}{2x^2} = \lim_{x\to\infty} \frac{5}{2} = \frac{5}{2}$.

If considering only the highest order terms does not appeal, one can instead divide the numerator and denominator by x^4 and reduce the limit using the known fact that $\lim_{x\to\pm\infty}\frac{1}{x^k}=0$ for positive k:

$$\lim_{x\to\infty}\frac{5x^4-2x^2+x}{2x^4+1}=\lim_{x\to\infty}\frac{5-\frac{2}{x^2}+\frac{1}{x^3}}{2+\frac{1}{x^4}}=\lim_{x\to\infty}\frac{5-0+0}{2+0}=\frac{5}{2}$$

3. **(4 points)** *Calculate* $\frac{d}{dt}\left(3\sqrt{t}-\frac{5}{t^3}\right)$.

   Rephrasing radicals and division, this expression is simply $\frac{d}{dt}\left(3t^{\frac{1}{2}}-5t^{-3}\right)$, which is amenable to power rule evaluation to become $\frac{3}{2}t^{-\frac{1}{2}}+15t^{-4}=\frac{3}{2\sqrt{t}}+\frac{15}{t^4}$.

   It is possible to differentiate $\frac{5}{t^3}$ using the quotient rule, but doing so is generally regarded as more difficult than using the power rule directly.

4. **(4 points)** *Given* $f(x)=3x^2-2$, *use the difference quotient to calculate* $f'(x)$.

$$\begin{aligned}
f'(x) &= \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}\\
&= \lim_{h\to 0}\frac{[3(x+h)^2-2]-(3x^2-2)}{h}\\
&= \lim_{h\to 0}\frac{3x^2+6xh+3h^2-2-(3x^2-2)}{h}\\
&= \lim_{h\to 0}\frac{6xh+3h^2}{h}\\
&= \lim_{h\to 0}6x+3h \text{ justified since } h\neq 0\\
&= 6x
\end{aligned}$$