1. **(14 points)** *Answer the questions below:*

   (a) **(5 points)** *Find* $\frac{d}{d\theta}\left[(\cos\theta)e^{\sec\theta}\right]$.
   Using the product rule:

$$\frac{d}{d\theta}\left[(\cos\theta)e^{\sec\theta}\right] = \left(\frac{d}{d\theta}\cos\theta\right)e^{\sec\theta}+(\cos\theta)\frac{d}{d\theta}\left(e^{\sec\theta}\right) = -\sin\theta e^{\sec\theta}+(\cos\theta)\frac{d}{d\theta}\left(e^{\sec\theta}\right)$$

   Finding the derivative of $e^{\sec\theta}$, however, will require the chain rule: if $y = e^u$ and $u = \sec\theta$, then $\frac{dy}{du} = e^u$ and $\frac{du}{d\theta} = \sec\theta\tan\theta$, so $\frac{dy}{d\theta} = e^u\sec\theta\tan\theta = e^{\sec\theta}sec\theta\tan\theta$. Thus:

$$\begin{aligned}\frac{d}{d\theta}\left[(\cos\theta)e^{\sec\theta}\right] &= -\sin\theta e^{\sec\theta} + (\cos\theta)\frac{d}{d\theta}\left(e^{\sec\theta}\right)\\ &= -\sin\theta e^{\sec\theta} + (\cos\theta)e^{\sec\theta}\sec\theta\tan\theta\\ &= e^{\sec\theta}(\tan\theta - \sin\theta)\end{aligned}$$

   The last line of the above is a purely cosmetic change, using grouping and canceling the product $\cos\theta\sec\theta$.

   (b) **(4 points)** *For* $g(t) = \arctan(t\ln t)$, *find* $g'(t)$.
   This is a chain rule problem: denoting $g(t)$ by y, we may say that $y = \arctan u$ and $u = t\ln t$. Then $\frac{dy}{du} = \frac{1}{1+u^2}$ and, using the product rule, $\frac{du}{dt} = 1\ln t + t\frac{1}{t} = t+1$. Thus,

$$g'(t) = \frac{dy}{dt} = \frac{1}{1+u^2}(t+1) = \frac{1}{1+(t\ln t)^2}(t+1) = \frac{t+1}{1+(t\ln t)^2}$$

   (c) **(5 points)** *Given that* $y = e^{\arcsin(t^2)}$, *find* $\frac{dy}{dt}$.
   This is a several-layers-deep chain-rule problem: Let $y = e^u$, $u = \arcsin v$, and $v = t^2$. Then $\frac{dy}{du} = e^u$, $\frac{du}{dv} = \frac{1}{\sqrt{1-v^2}}$, and $\frac{dv}{dt} = 2t$. Assembling all the pieces and putting things in terms of t:

$$\begin{aligned}\frac{dy}{dt} &= \frac{dy}{du}\frac{du}{dv}\frac{dv}{dt}\\ &= e^u\frac{1}{\sqrt{1-v^2}}2t\\ &= \frac{2t}{\sqrt{1-v^2}}e^{\arcsin v}\\ &= \frac{2t}{\sqrt{1-t^4}}e^{\arcsin(t^2)}\end{aligned}$$

2. **(6 points)** $2x^2 - 3xy - 6y^2 + 2x - 4y = -6$ *is the equation of a hyperbola.*

   (a) **(4 points)** *On this curve, find* $\frac{dy}{dx}$ *as an expression in* x *and* y.

We implicitly differentiate the equation, using the chain and product rules as appropriate, then gather terms:

$$\begin{aligned}
\frac{d}{dx}(2x^2 - 3xy - 6y^2 + 2x - 4y) &= \frac{d}{dx}(-6) \\
4x - 3\frac{d}{dx}(xy) - 6\frac{d}{dx}y^2 + 2 - 4\frac{dy}{dx} &= 0 \\
4x - 3\left(\frac{d}{dx}x\right)y + 3x\left(\frac{d}{dx}y\right) - 6\frac{dy}{dx}\frac{d}{dy}y^2 + 2 - 4\frac{dy}{dx} &= 0 \\
4x - 3y - 3x\frac{dy}{dx} - 12\frac{dy}{dx}y + 2 - 4\frac{dy}{dx} &= 0 \\
4x - 3y + 2 + (-3x - 12y - 4)\frac{dy}{dx} &= 0 \\
(3x + 12y + 4)\frac{dy}{dx} &= 4x - 3y + 2 \\
\frac{dy}{dx} &= \frac{4x - 3y + 2}{3x + 12y + 4}
\end{aligned}$$

(b) **(2 points)** *Find the equation of the tangent line to this curve at* $(1, -2)$*.*

When $x = 1$ and $y = -2$, we know from the above formula for $\frac{dy}{dx}$ that

$$\frac{dy}{dx} = \frac{4(1) - 3(-2) + 2}{3(1) + 12(-2) - 4} = \frac{12}{-17}$$

so the tangent line to the curve at $(1, -2)$ has slope $\frac{-12}{17}$, and thus has equation, in point-slope form, of

$$(y + 2) = \frac{-12}{17}(x - 1)$$