1. **(6 points)** *Liquids in microgravity tend to form perfect spheres, and a sphere or radius r has volume $\frac{4}{3}\pi r^3$. A scientist on the international space station is sipping at a sphere of orange juice, drinking 3 cubic centimeters per second. How quickly is the radius of her drink changing if the radius is currently 8 centimeters?*

   Let us label the radius of the beverage as r, and its volume as V, so that $V = \frac{4}{3}\pi r^3$. What we desire is the rate of change of the radius; that is to say, $\frac{dr}{dt}$. We differentiate the above relationship with respect to t to get

$$\frac{d}{dt}V = \frac{d}{dt}\frac{4}{3}\pi r^3$$
$$\frac{dV}{dt} = \frac{dr}{dt}\frac{d}{dr}\frac{4}{3}\pi r^3$$
$$\frac{dV}{dt} = 4\pi\frac{dr}{dt}r^2$$
$$\frac{\frac{dV}{dt}}{4\pi r^2} = \frac{dr}{dt}$$

   The problem statement tells us that $\frac{dV}{dt} = -3$, and that at present $r = 8$, so $\frac{dr}{dt} = \frac{-3}{4\pi\cdot 8^2} = \frac{-3}{256\pi}$ so the radius of the sphere is decreasing at a rate of $\frac{3}{256\pi}$ centimeters per second.

2. **(7 points)** *Approximate the following quantites as accurately as possible using a well-chosen linear approximation method. Your answer should be expressed as a decimal.*

   (a) **(4 points)** $(0.95)^6$.
   0.95 is very close to the value 1, where the function $f(x) = x^6$ and its derivative $f'(x) = 6x^5$ are easy to calculate. Thus we will use the approximation:

$$f(0.95) \approx f(1) + (0.95 - 1)f'(1) = 1^6 + (-0.5)(6 \cdot 1^5) = 1 - 0.5 \cdot 6 = 0.7$$

   This is a pretty good approximation: the actual value of 0.95^6 is exactly 0.735091890625.

   (b) **(3 points)** $\sin 0.03$.
   0.03 is very close to the vale 0, where the function $f(x) = \sin x$ and its derivative $f'(x) = \cos x$ are easy to calculate. Thus we will use the approximation

$$f(0.03) \approx f(0) + (0.03 - 0)f'(0) = 0 + (0.03 - 0)1 = 0.03$$

   This is a pretty good approximation: the actual value of $\sin 0.03$ is approximately 0.0299955.

3. **(7 points)** *Consider the function $f(x) = 2x^3 + 3x^2 - 36x + 4$.*

   (a) **(3 points)** *Find the critical points of this function.*
   $f'(x) = 6x^2 + 6x - 36 = 6(x^2 + x - 6) = 6(x+3)(x-2)$. Critical points of $f(x)$ are those points at which $f'(x)$ is nonexistent or zero. Since $f'(x)$ is a polynomial, it is never nonexistent, but from the factorization shown here, it is clear that $f'(x) = 0$ when $x = -3$ or $x = 2$.

(b) **(2 points)** *Find the global maxima and minima of this function if they exist, or explain why not, if not.*

$f(x)$ is a polynomial of degree 3; we expect that $f(x)$ will increase without bound as x increases, and decrease without bound as $f(x)$ decreases; or, in symbols, we would say that $\lim_{x\to+\infty} f(x) = +\infty$ and $\lim_{x\to-\infty} f(x) = -\infty$. Since $f(x)$ increases and decreases without bound, it has no maximum or minimum.

(c) **(2 points)** *Find the maxima and minima of this function on the interval* $[0, 4]$ *if they exist, or explain why not, if not.*

We have three prospective extrema: the critical point $x = 2$, the interval endpoint $x = 0$, and the interval endpoint $x = 4$. We do not include the critical point $x = -3$ because it is not in the interval under consideration. Evaluating the function $f(x)$ at these three points, we find that $f(0) = 4$, $f(2) = -40$, and $f(4) = 36$. So $x = 2$ is the minimum on this interval, and $x = 4$ is the maximum.