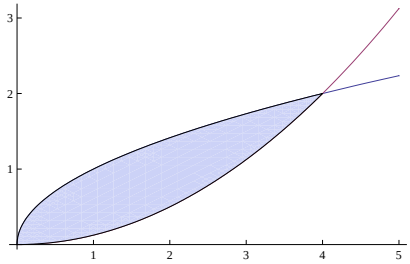
This test is closed-book and closed-notes. No calculator is allowed for this test. For full credit show all of your work (legibly!), unless otherwise specified.

The problems are in no particular order, and it is suggested that you look at all of them before beginning to answer any.

1. **(15 points)** The region shown below is the area between the curves $y = \sqrt{x}$ and $y = \frac{1}{8}x^2$. Find the center of mass of this region.



2. **(10 points)** Consider the curve $y = x^3 + 1$ between the points $(2, 9)$ and $(3, 28)$.

   (a) **(6 points)** Construct, but do not evaluate, an integral representing the length of this curve.

   (b) **(2 points)** Construct, but do not evaluate, an integral representing the surface area of the surface produced by rotating this curve around the line $y = 1$.

   (c) **(2 points)** Construct, but do not evaluate, an integral representing the surface area of the surface produced by rotating this curve around the y-axis.

3. **(15 points)** Consider the function $f(x) = \begin{cases} 0 \text{ for } x < 2 \\ \dfrac{64}{x^5} \text{ for } x \geq 2 \end{cases}$

   (a) **(6 points)** Prove that $f(x)$ is a probability distribution function.

   (b) **(3 points)** For a random variable X described by the above probability distribution function, find $P(X < 4)$.

   (c) **(6 points)** For a random variable X described by the above probability distribution function, find the average value of X.

4. **(15 points)** Perform the approximations shown below.

   (a) **(5 points)** Using Simpson's rule with $n = 4$, approximate $\int_0^8 \frac{1}{x^3+1} dx$. You need not arithmetically simplify your result.

   (b) **(10 points)** Using Euler's method with a step size of 2, approximate the value of y when $x = 16$ if $y = 5$ when $x = 10$ and $\frac{dy}{dx} = \frac{x}{y}$.

5. **(15 points)** Evaluate the following integrals, or if they cannot be evaluated, explain why not.

   (a) **(8 points)** $\int_{-4}^{3} \frac{1}{(x-2)^4} dx$

   (b) **(7 points)** $\int_{-\infty}^{6} xe^{-x^2} dx$

6. **(20 points)** Answer the following questions about the differential equation $\frac{dy}{dx} = 3x^2\sqrt{1-y^2}$

   (a) **(5 points)** Demonstrate without explicitly solving the differential equation that $y = \sin(x^3)$ is a solution.

   (b) **(10 points)** Find the general solution of the differential equation.

   (c) **(5 points)** Using your general solution, find a solution to the differential equation subject to the initial condition that $y = 1$ when $x = 0$.

7. **(10 points)** Find the general solution to the differential equation $\frac{dy}{dx} + 5y = 2x$.