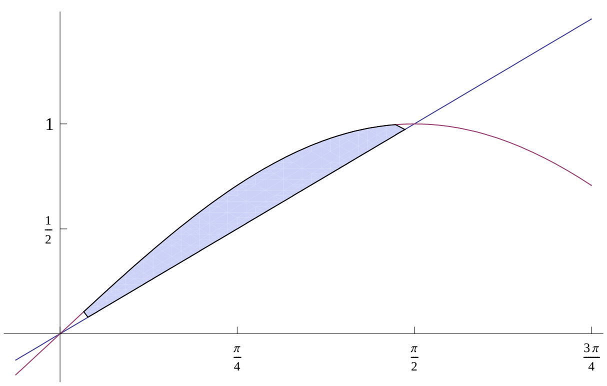
1. **(8 points)** *Calculate the following integrals:*

   (a) **(4 points)** $\int_0^{\pi/2} (\sin\theta \cos^3\theta) d\theta$.
   The substitution in this integral is suggested by two notable features: first, the existence of the composition $(\cos\theta)^3$ in the integrand, and the fact that the integrand involves both a particular function $(\cos\theta)$ and its derivative $(-\sin\theta)$. Based on both these hints it seems that the substitution $u = \cos\theta$ would be best for solving this problem. Then $du = \left(\frac{d}{d\theta}\cos\theta\right) dx = -\sin\theta d\theta$, so the phrase $\sin\theta d\theta$ appearing in the integral above can be rewritten as $(-du)$. Thus the above integral becomes

$$\int_{\theta=0}^{\theta=\pi/2} u^3(-du) = \frac{-u^4}{4}\Bigg]_{\theta=0}^{\theta=\pi/2} = \frac{-\cos^4\theta}{4}\Bigg]_0^{\pi/2} = \frac{-\cos^4\frac{\pi}{2} + \cos^4 0}{4} = \frac{1}{4}$$

   (b) **(4 points)** $\int 5e^{3t} + 2dt$.
   We start by separating the sum into two distinct integrals, since 2 is easily integrated: $\int 5e^{3t} + 2dt = \int 5e^{3t}dt + 2t$. Then, the remaining term can be solved either with an explicit substitution $u = 3t$ or the implicit rule for linear substitution; in either case, a $\frac{1}{3}$ term is introduced, so the result is $\frac{5}{3}e^t + 2t + C$.

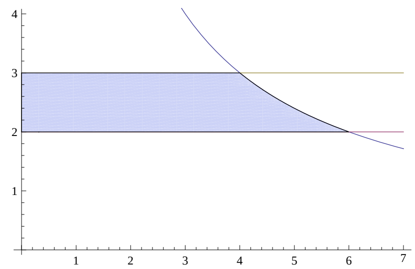
2. **(8 points)** *Calculate the area in the region between* $y = \frac{2x}{\pi}$ *and* $y = \sin x$ *shown below:*



As shown on the graph, this region begins on the left at $x = 0$ and ends on the right at $x = \frac{\pi}{2}$. Furthermore, $y = \sin x$ is the higher of the two functions throughout its length, so all necessary information to construct the integral is on hand:

$$\int_0^{\pi/2} \sin x - \frac{2}{\pi}xdx = -\cos x - \frac{1}{\pi}x^2\Bigg]_0^{\pi/2} = (-0 - \frac{1}{\pi}\frac{\pi^2}{4}) - (-1 - 0) = 1 - \frac{\pi}{4}$$

3. **(8 points)** *Calculate the volume of the solid produced by rotating the region bounded by the y-axis,* $y = \frac{12}{x}$, $y = 2$, *and* $y = 3$ **around the y-axis**.



Using the disc method with respect to the y-axis, we'll need to integrate with respect to y (i.e., working bottom to top). Since our boundaries are $y = 2$ and $y = 3$, we will integrate from 2 to 3. At a particular height y, the right boundary is $y = \frac{12}{x}$, which gives us $x = \frac{12}{y}$, so the radius of each slice is $\frac{12}{y}$. Thus, our integral is:

$$\int_2^3 \pi \left(\frac{12}{y}\right)^2 dy = \int_2^3 144\pi y^{-2} = -144\pi y^{-1}\Big]_2^3 = -\frac{144}{3}\pi + \frac{144}{2}\pi = (72 - 48)\pi = 24\pi$$

It is possible, but actually considerably more difficult, to perform this integral with respect to x and use the cylidrical-shell method.

4. **(2 point bonus)** *Calculate the volume of the solid produced by rotating the region shown in the above problem around the x-axis.*

If we wish to use the disc/washer method, we would integrate with respect to x (a shell approach is possible as well). Here, the shape goes from $x = 0$ to $x = 6$, but changes shape radically at $x = 4$, so we would actually construct two separate integrals: wehn x is between 0 and 4, the outer radius is 3 and the inner radius is 2, while for x between 4 and 6, the outer radius is $\frac{12}{x}$ and the inner radius is 2. Thus, to calculate the volume of the entire figure, we add the two integrals:

$$\int_0^4 \pi(3^2 - 2^2)dx + \int_4^6 \pi \left(\frac{144}{x^2} - 2^2\right) dx$$