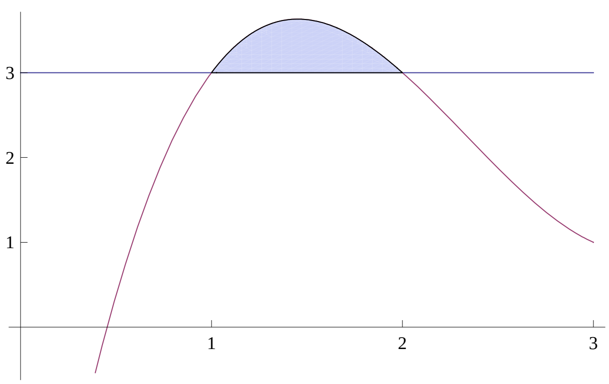
1. **(6 points)** *Evaluate* $\int(4x+2)\sin(2x)dx$ *via integration by parts.*

   Since this integral is a product of a polynomial and an integrable function, our parts decomposition will choose u to be the polynomial, and v the trigonometric function. So we have $u = 4x+2$ and $dv = \sin(2x)dx$; then $du = 4dx$ and $v = \frac{-1}{2}\cos(2x)$, using an implicit linear substitution. With these four quantities in hand, we may perform the integration by parts, and from then on proceed to complete the evaluation of the integral:

$$\begin{aligned}\int(4x+2)\sin(2x)dx &= (4x+2)\left(\frac{-1}{2}\cos(2x)\right) - \int\frac{-1}{2}\cos(2x)(4dx)\\ &= -(2x+1)\cos(2x) + 2\int\cos(2x)dx\\ &= -(2x+1)\cos(2x) + \sin(2x) + C\end{aligned}$$

2. **(6 points)** *Set up an evaluatable integral whose value is the volume of the solid produced by rotating the region bounded by* $y = x^3 - 7x^2 + 14x - 5$ *and* $y = 3$ **around the y-axis**. *It is* **not necessary to evaluate the integral**.



   Using horizontal cross-sections would be difficult or impossible in this case, since placing x in terms of y is extremely difficult for cubics. Instead, we will use vertical cross-sections, which, since we are rotating around the y-axis, will revolve into cylindrical shells. Our range of x-values in which the region in question exists can be seen on the shown figure to be $1 \leq x \leq 2$, so our integral prototype would be $\int_1^2 2\pi rh dx$, where r and h are the radius and height respectively of the shell produced by a cross-section at a particular x-value. The radius of such a shell is simply the horizontal distance of the cross-section from the y-axis, which can be seen to be x. The height of each shell is simply the height of the shaded region at the given x-value, which is the differences of the upper and lower functions: $(x^3 - 7x^2 + 14x - 5) - 3 = x^3 - 7x^2 + 14x - 8$. Putting these quantities into the integral, we get the form

$$\int_1^2 2\pi x(x^3 - 7x^2 + 14x - 8)dx$$

   which, if we needed to, could be solved fairly easily with nothing more involved than the power law (but which is not requested in the problem statement).

3. **(6 points)** *Evaluate the trig-substitution integral* $\int \frac{x^3}{\sqrt{x^2-4}}dx$

Since this integral contains $\sqrt{x^2-4}$, a trig substitution is suggested. We place x on the hypotenuse of a right triangle with marked angle θ and 2 on an adjacent side to get $\sqrt{x^2-4}$ as the length of the opposite side. Thus, using the definition of the trig ratios, $x = 2\sec\theta$ and $\sqrt{x^2-4} = 2\tan\theta$. Furthermore, differentiating our first substitution gives $dx = 2\sec\theta\tan\theta d\theta$. Using all these substitutions:

$$\int \frac{x^3}{\sqrt{x^2-4}}dx = \int \frac{(2\sec\theta)^3}{2\tan\theta}(2\sec\theta\tan\theta d\theta) = \int 8\sec^4\theta d\theta$$

This is an integral with an even number of secants, so the transformation to solve this trigonometric integral is to isolate a $sec^2\theta$ term and convert the rest to tangent terms, which we plan to use in a u-substitution. We do so with the identity $\sec^2\theta = \tan^2\theta + 1$, which gives us

$$\int 8\sec^4\theta d\theta = \int 8(1+\tan^2\theta)\sec^2\theta d\theta$$

and now we use a u-substitution: $u = \tan\theta$, $du = \sec^2\theta d\theta$. Then the above becomes $\int 8(1+u^2)du = 8u + \frac{8}{3}u^3 + C$. Replacing u with $\tan\theta$, and then $2\tan\theta$ with $\sqrt{x^2-4}$, we get

$$8\tan\theta + \frac{8\tan^3\theta}{3} + C = 4\sqrt{x^2-4} + \frac{\sqrt{x^2-4}^3}{3} + C$$

4. **(6 points)** *Using partial fraction decomposition, evaluate the integral* $\int \frac{2x+1}{x^2+6x+9}dx$.

Since $x^2 + 6x + 9 = (x+3)^2$, the appropriate decomposition is

$$\frac{2x+1}{x^2+6x+9} = \frac{A}{x+3} + \frac{B}{(x+3)^2}$$

which, on multiplying by the common denominator, yields

$$2x + 1 = A(x+3) + B$$

which, on comparison of linear terms requires that $2 = A$, and on comparing constant terms, requires that $1 = 3A + B$. Since $2 = A$, the latter equation can be simplified to $-5 = B$. Thus, we have the decomposition

$$\int \frac{2x+1}{x^2+6x+9}dx = \int \frac{2}{x+3} - \frac{5}{(x+3)^2}dx = 2\ln|x+3| + \frac{5}{x+3} + C$$