1. **(12 points)** *Showing all necessary steps, evaluate the two following integrals, or if they cannot be evaluated, explain why:*

   (a) **(6 points)** $\int_{-5}^{1} \frac{1}{\sqrt{x+4}} dx$.

   This integral is in fact discontinuous in the entire interval $[-5, -4]$, so cannot be evaluated and is thus divergent within the boundaries of what we have learned in this course; if, however, imaginary numbers are considered as valid elements of the range of a function, we run into a simple infinite discontinuity at -4, thus, the following improper-integral splice is required:

$$\begin{aligned}
\int_{-5}^{1} \frac{1}{\sqrt{x+4}} dx &= \int_{-5}^{-4} \frac{1}{\sqrt{x+4}} dx + \int_{-4}^{1} \frac{1}{\sqrt{x+4}} dx \\
&= \lim_{b \to -4^-} \int_{-5}^{b} \frac{1}{\sqrt{x+4}} dx + \lim_{a \to -4^+} \int_{a}^{1} \frac{1}{\sqrt{x+4}} dx \\
&= \lim_{b \to -4^-} \left. \frac{(x+4)^{1/2}}{1/2} \right]_{-5}^{b} + \lim_{a \to -4^+} \left. \frac{(x+4)^{1/2}}{1/2} \right]_{a}^{1} \\
&= \lim_{b \to -4^-} 2(b+4)^{1/2} - 2(-5+4)^{1/2} + \lim_{a \to -4^+} 2(1+4)^{1/2} - 2(a+4)^{1/2} \\
&= \sqrt{0} - \sqrt{-1} + \sqrt{5} - \sqrt{0}
\end{aligned}$$

   which is an unusual answer — as might be expected, inasmuch as it includes the peculiar (and in real numbers, uncalculatable) phrase $\sqrt{-1}$. This integral could be recognized as divergent when calculated over the real nubmers, or taken to be $\sqrt{5} - i$ if calculated over the complex plane.

   (b) **(6 points)** $\int_{-\infty}^{-2} \frac{1}{t^2} dt$.

   This is an improper integral which must be rephrased as the limit of definite integral with lower bound decreasing without boud, as such:

$$\int_{-\infty}^{-2} \frac{1}{t^2} dt = \lim_{a \to -\infty} \int_{a}^{-2} \frac{1}{t^2} dt = \lim_{a \to -\infty} \left. \frac{-1}{t} \right]_{a}^{-2} = \lim_{a \to -\infty} \frac{-1}{-2} - \frac{-1}{a} = \frac{-1}{-2} - 0 = \frac{1}{2}$$

2. **(4 points)** *Based on the following table, estimate $\int_0^2 f(x) dx$ using both the trapezoidal rule and Simpson's rule. Label which is which. You need not arithmetically simplify your result.*

x	0.0	0.5	1.0	1.5	2.0
$f(x)$	3	6	2	0	-2

As can be clearly seen in the table, the sample points are x-values spaced evenly at a distance of 0.5, so the x-step δx is equal to 0.5. Thus, simple application of known rules yields that a trapezoidal-rule approximation of this integral is

$$\frac{0.5}{2} (1 \cdot 3 + 2 \cdot 6 + 2 \cdot 2 + 2 \cdot 0 + 1 \cdot -2)$$

which evaluates to $\frac{17}{4}$. Likewise, the Simpson's rule approximation would be

$$\frac{0.5}{3}(1 \cdot 3 + 4 \cdot 6 + 2 \cdot 2 + 4 \cdot 0 + 1 \cdot -2)$$

which evaluates to $\frac{29}{6}$, which is not particularly close to $\frac{17}{4}$, but four intervals make for very poor estimation, so two estimates may be quite different.

3. **(8 points)** *Answer the following questions about the curve given by the equation* $y = \sin x$ *between* $x = \frac{\pi}{4}$ *and* $x = \frac{\pi}{2}$.

   (a) **(4 points)** *Set up, but do not evaluate, an integral representing the length of the aforementioned curve.*

   The arc-length differential is given by $ds = \sqrt{1 + [f'(x)]^2}dx$, so in this case, measuring the length of the curve $f(x) = \sin x$ between $x = \frac{\pi}{4}$ and $x = \frac{\pi}{2}$, our integral would be:

$$\int_{\pi/4}^{\pi/2} \sqrt{1 + \left(\frac{d}{dx}\sin x\right)^2} dx = \int_{\pi/4}^{\pi/2} \sqrt{1 + \cos^2 x} dx$$

   (b) **(2 points)** *Set up, but do not evaluate, an integral representing the surface area of the solid produced by rotating the aforementioned curve about the* x*-axis.*

   As originally written, this problem used the word "volume" in place of the word "surface area". A rotating curve has no volume; the charitable intepretation of such a phrase would be that what is meant is the rotation of the region *under the curve*, in which case a disc method would yield $\int_{\pi/4}^{\pi/2} \pi \sin^2 x dx$.

   Answering the question as intended, however, would involve arcs given, as shown in part (a), to be of differential length $\sqrt{1 + \sin^2 x}dx$, being spun around circles of radius $\sin x$, since the vertical distance to the x-axis from the point $(x, \sin x)$ is simply $\cos x$; thus the differential area traced out is $2\pi \sin x \sqrt{1 + \cos^2 x}dx$, so the integral to compute the total surface area is

$$\int_{\pi/4}^{\pi/2} 2\pi \sin x \sqrt{1 + \cos^2 x} dx$$

   (c) **(2 points)** Set up, but do not evaluate, an integral representing the surface area of the solid produced by rotating the aforementioned curve about the y-axis.

   As originally written, this problem used the word "volume" in place of the word "surface area". A rotating curve has no volume; the charitable intepretation of such a phrase would be that what is meant is the rotation of the region *under the curve*, in which case a cylindircal-shells method would yield $\int_{\pi/4}^{\pi/2} 2\pi x \sin x dx$.

   Answering the question as intended, however, would involve arcs given, as shown in part (a), to be of differential length $\sqrt{1 + \cos^2 x}dx$, being spun around circles of radius x, since the horizontal distance to the y-axis from the point $(x, \cos x)$ is simply x; thus the differential area traced out is $2\pi x\sqrt{1+\cos^2 x}dx$, so the integral to compute the total surface area is

$$\int_{\pi/4}^{\pi/2} 2\pi x\sqrt{1+\cos^2 x}dx$$