

This test is closed-book and closed-notes. No calculator is allowed for this test. When answering the questions you are only allowed to use tools and techniques that we have discussed up to this point, i.e. no advanced differentiation techniques. For full credit show all of your work (legibly!), unless otherwise specified.

For the purposes of this exam, “familiar functions” includes all arithmetic operations as well as trigonometric functions, inverse trigonometric functions, base 10 logarithms, and natural logarithms.

The problems are in no particular order, and it is suggested that you look at all of them before beginning to answer any.

**Do not open this packet until instructed to begin!**

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1. **(8 points)** Answer the following questions for the function  $f(t) = -5 \cos(3\pi t)$ .
  - (a) **(3 points)** What are its domain and range?
  
  
  
  
  
  
  
  
  
  
  - (b) **(2 points)** Is it odd, even, both, or neither?
  
  
  
  
  
  
  
  
  
  
  - (c) **(3 points)** What are its amplitude and period?
  
  
  
  
  
  
  
  
  
  
2. **(8 points)** Given the function  $g(x) = \frac{(x+1)(x^2+1)}{(x+1)(4x-2)(3x-1)}$ , answer the following questions preparatory to sketching the functions.
  - (a) **(2 points)** What is the domain of the function?
  
  
  
  
  
  
  
  
  
  
  - (b) **(2 points)** What are all the zeroes of the function?
  
  
  
  
  
  
  
  
  
  
  - (c) **(2 points)** What are all the vertical asymptotes of the function?
  
  
  
  
  
  
  
  
  
  
  - (d) **(2 points)** Describe, either in words or symbolically, the long-term behavior of the function in each direction.

3. **(8 points)** Let  $g(s) = \frac{-2s^2+5s-3}{s-1}$ .

(a) **(1 point)** Find  $\lim_{s \rightarrow 1} g(s)$ .

(b) **(4 points)** Using epsilon-delta methods, justify your result above.

(c) **(3 points)** State the mathematical definition of the expression  $\lim_{x \rightarrow a^-} f(x) = L$ .

4. **(8 points)** Let  $f(x) = \begin{cases} x^2 & \text{if } x \leq 1 \\ 3x + a & \text{if } 1 < x \leq 4. \\ \sqrt{bx} & \text{if } x > 4 \end{cases}$ .

What choices of  $a$  and  $b$  will make this function continuous?

5. **(8 points)** Let  $f(x) = x^2 - 4x$ .

(a) **(6 points)** Using the difference quotient, find  $f'(x)$ .

(b) **(2 points)** Find the equation of the tangent line to  $f(x)$  at the point  $(-1, 5)$ .

6. **(8 points)** Rex Tyler consumes 200mg of the drug Miraclo in the morning. 18% of the drug is eliminated from his system every hour thereafter.

(a) **(3 points)** Create a function  $f(t)$  to describe the quantity of Miraclo still in his body  $t$  hours after he takes the pill.

(b) **(5 points)** In order to be able to sleep, he must have less than 50mg of the drug in his body. How soon will this occur?

7. **(8 points)** Evaluate the following limits; when a limit can not be evaluated, explain why or describe its behavior.

(a) **(2 points)**  $\lim_{u \rightarrow 3^+} \sqrt{9 - u^2}$

(b) **(2 points)**  $\lim_{\alpha \rightarrow 0} \frac{4\alpha^2 - 2\alpha}{\alpha}$

(c) **(2 points)**  $\lim_{x \rightarrow -\infty} \frac{x^3 - 3x^2 + 2x - 1}{20x^2}$

(d) **(2 points)**  $\lim_{\theta \rightarrow \frac{\pi}{3}^-} \frac{\cos \theta}{\theta}$

8. **(4 point bonus)** Simplify the expression  $\tan(\arcsin x)$ .