

This test is closed-book and closed-notes. No calculator is allowed for this test. For full credit show all of your work (legibly!), unless otherwise specified.

For the purposes of this exam, “familiar functions” includes all arithmetic operations as well as trigonometric functions, inverse trigonometric functions, base 10 logarithms, and natural logarithms. Any direct differentiation is considered complete as soon as all derivatives are performed; algebraic simplification is not generally necessary.

The problems are in no particular order, and it is suggested that you look at all of them before beginning to answer any.

Do not open this packet until instructed to begin!

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1. **(8 points)** Answer the following questions related to the shape of the graph of $f(x) = 4x^3 + 6x^2 - 9x - 17$.

(a) **(2 points)** What are its critical points, and is each a local maximum, a local minimum, or neither?

(b) **(3 points)** Where is it concave up? Where is it concave down? Does it have any points of inflection?

(c) **(3 points)** Where is it increasing? Where is it decreasing?

2. **(8 points)** Find approximations to the following values using appropriate linearizations.

(a) **(4 points)** Find a rational approximation to $\sqrt[3]{27.04}$.

(b) **(4 points)** Find an accurate decimal approximation of $\frac{1}{993}$.

3. **(8 points)** A bottle of Unicum is taken out of a 20°F freezer and set on a table in a 65°F room. It has warmed up to 35°F after 10 minutes.

(a) **(4 points)** Produce a function $T(t)$ modeling the bottle's temperature t minutes after it has been taken out of the freezer.

(b) **(2 points)** How quickly is the bottle's temperature changing after it has been out of the freezer for 5 minutes?

(c) **(2 points)** The optimal consumption temperature is 45°F . How many minutes should the bottle be left outside the freezer before it is ready to serve?

4. **(8 points)** We are planning to design a rectangular, fenced garden plot. The front of the garden should be unfenced; the back needs a picket fence, and the sides should be fenced with chain-link. Chain-link fence costs \$8 per foot; a picket fence costs \$10 per foot. We have \$320 to spend on fencing. What dimensions should we choose for our garden to maximize its area?
5. **(8 points)** Hiro is motorcycling west at 60 mph from a point 120 miles east of the Black Sun, while Raven is 50 miles north of the Black Sun, driving northwards at 30 mph.
- (a) **(4 points)** Are they growing closer together or further apart, and at what speed are they doing so?
- (b) **(4 points)** In an hour, will they be growing closer together or further apart, and at what speed will they be doing so?

6. (8 points) Evaluate the following limits; if they cannot be evaluated, show why not.

(a) (2 points) $\lim_{x \rightarrow \infty} \frac{e^x - 5x}{10e^x + 3x^2}$

(b) (2 points) $\lim_{y \rightarrow 1} \frac{y^3 - y}{\arctan y}$

(c) (2 points) $\lim_{x \rightarrow -\infty} x e^{-x^2}$

(d) (2 points) $\lim_{\theta \rightarrow 0} \frac{\sin \theta}{\theta^2}$

7. (4 point bonus)

(a) (2 points) Prove that every continuous even function has at least one extremum.

(b) (2 points) Prove that every nonconstant odd function with a finite number of extrema has an even number of extrema.