

1. **(8 points)** Given the function $g(x) = \frac{-2(x^2-2x-3)(x^2+2x+4)}{(x-3)(3x+1)(x-4)}$, answer the following questions preparatory to sketching the functions.

(a) **(2 points)** What is the domain of the function?

(b) **(2 points)** What are all the zeroes of the function?

(c) **(2 points)** What are all the asymptotes of the function?

(d) **(2 points)** Describe, either in words or symbolically, the long-term behavior of the function in each direction.

2. **(8 points)** Let $f(x) = \frac{4}{x}$.

(a) **(6 points)** Using the difference quotient, find $f'(x)$.

(b) **(2 points)** Find the equation of the tangent line to $f(x)$ at the point $(4, \frac{1}{4})$.

3. **(8 points)** Risembool has a population of 300 at present. It is expected to grow by 10% over the course of the next 5 years, and continue to grow at this rate.

(a) **(3 points)** Create a function $f(t)$ which yields the population t years from now.

(b) **(5 points)** Using your function, determine how many years it will take for the population to reach 400.

4. **(8 points)** Let $f(x) = \frac{(4x^2-36)}{(x+3)}$.

(a) **(1 point)** Find $\lim_{x \rightarrow -3} f(x)$.

(b) **(4 points)** Using epsilon-delta methods, justify your result above.

(c) **(3 points)** State the mathematical definition of the expression $\lim_{x \rightarrow -\infty} f(x) = +\infty$.

5. **(8 points)** Answer the following questions for the function $h(\theta) = 9 \sin\left(\frac{\pi}{7}t\right) + 3$.

(a) **(2 points)** What are its domain and range?

(b) **(2 points)** Is it odd, even, both, or neither?

(c) **(2 points)** What are its amplitude and frequency?

(d) **(2 points)** Is it invertible? If so, what is its inverse, and if not, why not?

6. **(8 points)** Evaluate the following limits; when a limit can not be evaluated, explain why or describe its behavior.

(a) **(2 points)** $\lim_{h \rightarrow 0} \frac{|h|}{h}$

(b) **(2 points)** $\lim_{t \rightarrow \infty} 2 \arctan(-t)$

(c) **(2 points)** $\lim_{r \rightarrow 1^+} \frac{\sqrt{r-1}}{r}$

(d) **(2 points)** $\lim_{x \rightarrow -\infty} \frac{3x^3 - 2x}{x^5 + 1}$

7. **(8 points)** Consider $g(x) = |x - 5| + \sec x$.

(a) **(2 points)** What is its domain?

(b) **(2 points)** Where is it continuous?

(c) **(2 points)** Where is it differentiable?

(d) **(2 points)** Prove that there is an x such that $g(x) = 6$.

8. **(4 point bonus)** Suppose functions f and g are inverses, and $f(a) = b$. What is the relationship between $f'(a)$ and $g'(b)$, and why does that relationship hold?