1. **(12 points)** Consider the function $g(x) = \frac{x+4}{x^2+3x}$.

   (a) **(5 points)** Identify zeroes, vertical asymptotes, and long-term behavior on both sides of this function. Clearly label which is which.

   (b) **(5 points)** Identify the critical points of this function, and indicate whether each is a local maximum, local minimum, or neither.

   (c) **(2 points)** Which if any of the critical points identified above are global maxima or global minima? Show work or explain.

2. **(12 points)** Answer the following questions:

   (a) **(4 points)** Find $\int \frac{d}{dx}\left(x^2\sqrt{1-x^2}\right)dx$.

   (b) **(4 points)** Given $h(x) = e^{\arctan(x^3)}$, find $h'(x)$.

   (c) **(4 points)** Find $\frac{d}{ds}\sin(se^s)$.

3. **(12 points)** A spherical balloon is being inflated by an air compressor which pumps 10 cubic centimeters of air per second. Note that the volume of a sphere of radius r is $\frac{4}{3}\pi r^3$.

   (a) **(9 points)** How quickly is the radius of the balloon growing when the balloon has a radius of 4 centimeters?

   (b) **(3 points)** How quickly is the radius of the balloon growing when the balloon has a radius of 10 centimeters?

4. **(12 points)** Let $f(x) = \frac{e^x}{x}$.

   (a) **(5 points)** Where is $f(x)$ increasing? Where is it decreasing?

   (b) **(7 points)** Determine $f(x)$'s concavity and identify points of inflection.

5. **(12 points)** The equation $x^4 = 2x^2y - y^3$ describes a figure known as a *bow curve*.

   (a) **(9 points)** Find a formula on this bow curve for $\frac{dy}{dx}$ in terms of x and y.

   (b) **(3 points)** Find the equation of the tangent line to the bow curve at the point $(4, -8)$.

6. **(12 points)** Determine the following limits.

   (a) **(4 points)** Evaluate $\lim_{\theta \to 0} \frac{1-\cos\theta}{\theta^2}$ or demonstrate that it cannot be evaluated.

   (b) **(4 points)** Evaluate $\lim_{x \to \infty} \frac{x \ln x}{x^3}$ or demonstrate that it cannot be evaluated.

   (c) **(4 points)** *Using the difference quotient*, find the derivative with respect to x of $f(x) = 5x^2 + 2x - 10$. You may not use L'Hôpital's rule for this problem.

7. **(12 points)** Evaluate the following integrals:

   (a) **(4 points)** $\int \frac{e^{\sqrt{x}}}{\sqrt{x}} dx$.

   (b) **(4 points)** $\int_0^{\pi/6} \frac{\tan x}{\cos x} dx$.

   (c) **(4 points)** $\int_0^1 e^x \sin(e^x) dx$

8. **(12 points)** You have been asked to design a box for shipping. It must be exactly twice as long as it is wide, but you have been given no restrictions on its height. Material for the bottom and top of the crate costs \$3 per square foot, while the material for the sides costs \$1 per square foot. You have been given \$20 for your task. What dimensions will give you the largest volume possible?

9. **(12 points)** A casserole is heated to 160°F and then removed from the oven into a 70°F room. After half an hour, it has cooled to 100°F.

   (a) **(4 points)** Construct a function modeling the temperature of the casserole t hours after it is removed from the oven.

   (b) **(4 points)** How quickly is the casserole cooling one hour after it is removed from the oven?

   (c) **(4 points)** How long will it take the casserole to cool to 80°F?

10. **(12 points)** Answer the following questions about the function $h(x) = \ln(x^3 - 8)$.

(a) **(4 points)** What is the domain of $h(x)$?

(b) **(4 points)** What is the range of $h(x)$?

(c) **(4 points)** Where does the derivative of $h(x)$ exist?