For full credit show all of your work (legibly!), unless otherwise specified. This exam is closed-notes and calculators may not be used. Answers need not be completely reduced unless otherwise stated, and may be left in terms of sums, differences, products, quotients, exponents, factorials, binomial coefficients, and multinomial coefficients.

1. **(12 points)** Answer the following questions.

   (a) **(6 points)** How many arrangements of the word "MURMUR" are there in which no two consecutive letters are the same?

   (b) **(6 points)** We have 15 distinct books which we would like to divide among 3 indistinguishable boxes. How many ways are there to do so if we require that each box be nonempty?

1	
2	
3	
4	
(5)	
Σ	

2. **(12 points)** Suppose we are partitioning a number into no more than 2 parts each of size 2, 3, or 4. For instance, 8 has 3 such partitions: $4+4$, $4+2+2$, $3+3+2$.

   (a) **(6 points)** Determine a generating function for the number of unordered partitions of n into parts of size 2, 3, or 4, with no more than 2 parts of each size. You need not expand the generating function algebraically.

   (b) **(2 points)** What will the highest exponent appearing in the expansion of the above generating function be? What does it signify?

   (c) **(4 points)** Demonstrate that the transpose of a partition meeting the conditions described above will have fewer than 5 parts and no part of size larger than 6.

3. **(12 points)** We have a row of several demarked small parking spaces on the side of the street into which we place vehicles of various types. A compact, a sedan, or a smartcar fits into a single space. An SUV, a van, a station wagon, or a pickup truck requires two spaces.

   (a) **(6 points)** Find a recurrence relation, with initial conditions, describing the number of configurations of various types of cars filling n parking spaces.

   (b) **(6 points)** Solve the above-determined recurrence relation to determine a closed form for the number of configurations.

4. **(12 points)**

(a) **(4 points)** Find (but do not expand) an exponential generating function for the number of ternary sequences containing an even number of 0s, fewer than four 1s, and at least two 2s.

(b) **(4 points)** Find a generating function describing the number of ways to put n indistinguishable items into 5 distinct boxes, with at least one and no more than five items per box.

(c) **(6 points)** Using your generating function, find the number of ways to distribute thirteen items in the manner described in part (b).

5. **(6 point bonus)** Find a generating function for the number of partitions of n such that if the parititon contains a part of size k, it also contains an part of every size less than k (e.g., 6 can be so divided 4 ways, into only 1s: $1+1+1+1+1+1$, 1s and 2s: $1+1+1+1+2$ or $1+1+2+2$, or 1s, 2s, and 3s: $1+2+3$.)