For full credit show all of your work (legibly!), unless otherwise specified. This exam is closed-notes and calculators may not be used. Answers need not be completely reduced unless otherwise stated, and may be left in terms of sums, differences, products, quotients, factorials, permutations, and combinations.

1. **(12 points)** We play a game in which we roll a seven-sided die; we call a sequence of rolls "good" if it does not have two consecutive rolls of a number 3 or less (i.e. containing the subsequence 11, 12, 13 21, 22, 23, 31, 32, or 33).

   (a) **(6 points)** Find a recurrence relation and initial conditions for $a_n$, the number of good sequences of length $n$.

   (b) **(3 points)** Find a closed-form (non-recurrence) formula for $a_n$.

2. **(12 points)** We have fifteen tasks to distribute to our four distinct employees. We must give each employee between 2 and 5 tasks.

   (a) **(6 points)** How many ways can we distribute the tasks if all the tasks are considered to be identical?

   (b) **(6 points)** Describe a method for calculating the number of ways to distribute the tasks if all the tasks are considered to be distinct. You need not actually perform the calculation.

3. **(12 points)**

(a) **(4 points)** Determine a generating function for the number of unordered partitions of $n$ into parts of size 1, 2, or 5.

(b) **(4 points)** Determine a recurrence relation and initial conditions for the number of *ordered* partitions of $n$ into parts of size 1, 2, or 5. (e.g. $6 = 5 + 1$ and $6 = 1 + 5$ are considered distinct partitions of 6.

(c) **(4 points)** Determine a generating function for the number of unordered partitions of $n$ into an even number of parts.

4. **(12 points)**

   (a) **(6 points)** Determine the coefficient of $z^n$ in the power-series expansion of $(z^2+z^3)(z+z^2+z^3+z^4+z^5+\cdots)^3$.

   (b) **(6 points)** Using an exponential generating function, find the number of arrangements of 5 letters of the word TENNESSEE.

5. **(12 points)**

(a) **(4 points)** A number is *squarefree* if it is not divisible by any square number greater than 1. How many squarefree positive integers are there less than 100?

(b) **(4 points)** How many surjections are there from $A = \{a, b, c, d, e, f\}$ to $B = \{1, 2, 3\}$?

(c) **(4 points)** How many 6-digit numbers have at least one of 1, 2, or 3 as a digit?