

This test is closed-book and closed-notes. No calculator is allowed for this test. When answering the questions you are only allowed to use tools and techniques that we have discussed up to this point, i.e. no advanced differentiation techniques. For full credit show all of your work (legibly!), unless otherwise specified.

For the purposes of this exam, unreduced results, where necessary, should be phrased in terms of standard arithmetic operators, trigonometric functions, inverse trigonometric functions and natural logarithms.

The problems are in no particular order, and it is suggested that you look at all of them before beginning to answer any.

1. **(8 points)** A sample of phlebotinum-75 decays over time, losing 23% of its mass per year. We have just obtained a 20-gram sample of this element.

(a) **(3 points)** Create a function $f(t)$ to describe the expected mass of phlebotinum-75 remaining in the sample t years from now.

(b) **(5 points)** The sample is too small to be of further use to us after only 5 grams remain. How long will it take for this to happen?

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2. **(8 points)** Given the function $f(x) = \frac{(x^2+1)(x+1)}{(x-2)(x+2)(2x+3)}$, answer the following questions preparatory to sketching the functions.

(a) **(2 points)** What is the domain of the function?

(b) **(2 points)** Describe, either in words or symbolically, the long-term behavior of the function in each direction.

(c) **(2 points)** What are all the vertical asymptotes of the function?

(d) **(2 points)** What are all the zeroes of the function?

3. **(8 points)** Let $f(x) = \begin{cases} x^2 + 1 & \text{if } x \leq 1 \\ \sqrt{x + a} & \text{if } 1 < x \leq 6. \\ bx & \text{if } x > 6 \end{cases}$.

What choices of a and b will make this function continuous?

4. **(8 points)** Let $g(u) = \frac{2u^2-3u-5}{u+1}$.

(a) **(1 point)** Find $\lim_{u \rightarrow -1} g(u)$.

(b) **(4 points)** Using epsilon-delta methods, justify your result above.

(c) **(3 points)** State the mathematical definition of the statement $\lim_{x \rightarrow +\infty} f(x) = L$.

5. **(8 points)** Let $g(x) = -3x^2 + 7x - 2$.

(a) **(6 points)** *Using the difference quotient*, find $g'(x)$.

(b) **(2 points)** Find the equation of the tangent line to $g(x)$ at the point $(2, 0)$.

6. **(8 points)** Answer the following questions for the function $f(t) = -2\cos(6\pi t) + 3$.

(a) **(3 points)** What are its amplitude and period?

(b) **(3 points)** What are its domain and range?

(c) **(2 points)** Is it odd, even, both, or neither? Briefly justify your answer.

7. **(8 points)** Evaluate the following limits; when a limit can not be evaluated, explain why or describe its behavior.

(a) **(2 points)** $\lim_{t \rightarrow +\infty} \frac{2t^3 - 4t^2 + 7}{-t^4 + 5t^2 - 2}$

(b) **(2 points)** $\lim_{\theta \rightarrow \pi^-} \sin \theta$

(c) **(2 points)** $\lim_{x \rightarrow 3} \frac{x^2 - x - 6}{x - 3}$

(d) **(2 points)** $\lim_{r \rightarrow 3} \frac{r^3 - 1}{r - 3}$

8. **(4 point bonus)** Using the difference quotient (on the back of this sheet), find the derivative of the function $f(x) = \frac{1}{x^2}$.