This test is closed-book and closed-notes. No calculator is allowed for this test. For full credit show all of your work (legibly!), unless otherwise specified.

Answers may include all arithmetic operations, trigonometric functions, inverse trigonometric functions, and natural logarithms. Algebraic simplification of answers is unnecessary.

The problems are in no particular order, and it is suggested that you look at all of them before beginning to answer any.

1. **(8 points)** The *right strophoid* is a curve satisfying the equation $xy^2 + 5y^2 = 5x^2 - x^3$.

   (a) **(6 points)** Find a formula for $\frac{dy}{dx}$ on this curve.

   (b) **(2 points)** Find the equation of the tangent line to the curve at $(-3, 6)$.

2. **(8 points)** Calculate $\frac{d}{dx}\csc(x^2 \tan x)$.

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3. **(8 points)** An air compressor which delivers 15 cubic feet per minute is inflating a spherical balloon. The volume of a sphere of radius $r$ is $\frac{4}{3}\pi r^3$.

   (a) **(6 points)** How quickly is the balloon's radius increasing when it has a radius of 5 feet?

   (b) **(2 points)** How quickly is the balloon's radius increasing when the radius is ten feet?

4. **(8 points)** Find an equation of the tangent line to the curve $y = \frac{x+1}{x-1}$ at $(2, 3)$.

5. **(8 points)** If $f(x) = \frac{e^{\arcsin x}}{\ln x}$, then find $f'(x)$.

6. **(8 points)** Answer the following derivative-related questions.

   (a) **(2 points)** Find $\frac{d}{dt}\frac{\sqrt{t}}{\cos t}$.

   (b) **(4 points)** If $y = \arctan\sqrt{e^x}$, find $\frac{dy}{dx}$.

   (c) **(2 points)** If $f(s) = s^3 \sec s$, find $f'(s)$.

7. **(8 points)** A piece of 1500°F lava lands at the base of a volcano on a sweltering 100°F day. After 5 minutes it has cooled to 1200°F.

   (a) **(4 points)** Produce a function $T(t)$ modeling the temperature of the lava fragment after $t$ minutes.

   (b) **(2 points)** How quickly is the fragment's temperature changing ten minutes after it lands on the beach?

   (c) **(2 points)** How long will the fragment take to cool to 500°F?

8. **(4 point bonus)** Find the following derivatives:

   (a) **(2 points)** What is $\frac{d}{dx}(x^x)$?

   (b) **(2 points)** The *Lambert W function* is a function $W(x)$ such that $W(x)e^{W(x)} = x$. Find a formula for $W'(x)$, using $W(x)$ itself as necessary.