This test is closed-book and closed-notes. No calculator is allowed for this test. For full credit show all of your work (legibly!), unless otherwise specified. Algebraic and trigonometric simplification of answers is generally unnecessary. The problems are in no particular order, and it is suggested that you look at all of them before beginning to answer any.

1. **(12 points)** Estimate the following values using appropriate linear approximations.

   (a) **(6 points)** Find a decimal approximation to $(1.03)^7$.

   (b) **(6 points)** Find a rational number approximately equal to $\sqrt[3]{7.95}$.

2. **(12 points)** Answer the following questions:

   (a) **(6 points)** Determine a region whose area is $\lim_{n\to\infty} \sum_{i=1}^{n} \left(\frac{5}{n}\right) \ln\left(2 + \frac{5i}{n}\right)$.

   (b) **(6 points)** Find the general antiderivative of $g(x) = 4 + \sqrt{x} - \sec x \tan x + \frac{2}{1+x^2}$.

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3. **(12 points)** Answer the following questions related to the shape of the graph of $f(x) = x^3 + 3x^2 - 24x + 6$.

   (a) **(4 points)** Where is it increasing? Where is it decreasing?

   (b) **(4 points)** What are its critical points, and is each a local maximum, a local minimum, or neither?

   (c) **(4 points)** Where is it concave up? Where is it concave down? Does it have any points of inflection?

4. **(12 points)** You are planning a design for a 1200-square-foot rectangular swimming pool, with a rectangle of paving around the entire pool. Around three sides of the pool you want to have a 3-foot paved strip; on the fourth side you want to have a 5-foot strip. What dimensions for the pool will minimize the necessary total area of the pool and poolside paving?

5. **(12 points)** Answer the following questions about approximation:

   (a) **(6 points)** Note that $\sqrt[5]{33}$ is a zero of the function $f(x) = x^5 - 33$. Choose an integer value of x_0 which is close to $\sqrt[5]{33}$. Use one step of Newton's method to develop x_1, a better rational approximation of $\sqrt[5]{33}$.

   (b) **(6 points)** Starting with an initial value of 1, use two iterations of Newton's method to approximate a zero of $f(x) = x^3 - 2x - 1$. Your answer need not be arithmetically simplified.

6. **(12 points)** Evaluate the following limits; if they cannot be evaluated, show why not.

   (a) **(3 points)** $\lim_{t\to+\infty} \frac{\ln t}{2t^2+1}$.

   (b) **(3 points)** $\lim_{\theta\to 0} \frac{\theta\cos\theta}{\sin\theta}$.

   (c) **(3 points)** $\lim_{x\to+\infty} (x+4)e^{-x}$.

   (d) **(3 points)** $\lim_{x\to 1} \frac{x^2-4x+4}{e^x}$.

7. **(4 point bonus)** We have two ways of roughly estimating $\sqrt[k]{n}$: linear approximation of the radical at some a near n, or one step of Newton's method on $f(x) = x^k - n$ with a well-chosen x_0. Prove that when $a = x_0^k$, these methods have the same answer.