This test is closed-book and closed-notes. No calculator is allowed for this test. For full credit show all of your work (legibly!), unless otherwise specified. For the purposes of this exam, all answers must be in terms of familiar functions. Algebraic and trigonometric simplification of answers is generally unnecessary.

The problems are in no particular order, and it is suggested that you look at all of them before beginning to answer any.

1. **(12 points)** Answer the following questions about approximation:

   (a) **(6 points)** Choose an integer value of x_0 approximating $\sqrt[3]{26}$ as well as possible. Use one step of Newton's method to develop a better rational approximation x_1.

   (b) **(6 points)** Starting with an initial value of 1, use two iterations of Newton's method to approximate a zero of $f(x) = x^4 - 5x + 1$. Your answer need not be arithmetically simplified.

1	
2	
3	
4	
5	
6	
Σ	

2. **(12 points)** Answer the following questions:

   (a) **(6 points)** Find the general antiderivative of the function $f(x) = 3x^3 + 4\sec^2 x - e^x + \frac{2}{x}$

   (b) **(6 points)** Determine a region whose area is $\lim_{n\to\infty} \sum_{i=1}^{n} \left(\frac{2}{n}\right) \sqrt[3]{1 + \frac{2i}{n}}$.

3. **(12 points)** Find approximations to the following values using appropriate linear approximations.

   (a) **(6 points)** Find a rational number approximately equal to $\sqrt{83}$.

   (b) **(6 points)** Find an approximation to $(1.998)^5$.

4. **(12 points)** You are designing a rectangular factory with 2400 square feet of floor space with walls on three sides and a loading dock on one side. Safety regulations demand that there be a 2-foot space between machinery and the walls, and a 4-foot space between machinery and the loading dock. What dimensions for the factory maximize the amount of space available for machinery?

5. **(12 points)** Evaluate the following limits; if they cannot be evaluated, show why not.

   (a) **(3 points)** $\lim_{x \to 0^+} x^2 \csc x$

   (b) **(3 points)** $\lim_{\theta \to \frac{\pi}{3}} \frac{\cos \theta}{\sin^2 \theta}$

   (c) **(3 points)** $\lim_{t \to \infty} \frac{e^t}{2t+1}$

   (d) **(3 points)** $\lim_{x \to -1} \frac{x^2+2x+1}{xe^x+e^{-1}}$

6. **(12 points)** Answer the following questions related to the shape of the graph of $f(x) = x^3 - 3x^2 - 45x + 2$.

   (a) **(4 points)** Where is it increasing? Where is it decreasing?

   (b) **(4 points)** What are its critical points, and is each a local maximum, a local minimum, or neither?

   (c) **(4 points)** Where is it concave up? Where is it concave down? Does it have any points of inflection?