This test is closed-book and closed-notes. No calculator is allowed for this test. For full credit show all of your work (legibly!), unless otherwise specified. For the purposes of this exam, all answers must be in terms of familiar functions. Algebraic and trigonometric simplification of answers is generally unnecessary.

1. **(12 points)** Evaluate the following integrals:

   (a) **(4 points)** $\int_{\pi/6}^{\pi/3} \frac{1}{\cos^2 x} dx$.

   (b) **(4 points)** $\int \frac{1}{x} \sec^2(\ln x) dx$.

   (c) **(4 points)** $\int_0^1 x^3 \sin(x^4 + 2) dx$

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2. **(12 points)** Answer the following questions about the function $f(x) = 3\sin\sqrt{x}$.

   (a) **(4 points)** What is the domain of $f(x)$?

   (b) **(4 points)** What is the range of $f(x)$?

   (c) **(4 points)** Where does the derivative of $f(x)$ exist?

3. **(12 points)** Let $f(x) = e^{-2x^2}$.

   (a) **(5 points)** Where is $f(x)$ increasing? Where is it decreasing?

   (b) **(7 points)** Determine $f(x)$'s concavity and identify points of inflection.

4. **(12 points)** We are planning to design a rectangular, fenced yard. The front of the garden should be unfenced; the back needs a picket fence, and the sides should be fenced with chain-link. Chain-link fence costs \$8 per foot; a picket fence costs \$10 per foot. We have \$640 to spend on fencing. What dimensions should we choose for our garden to maximize its area?

5. **(12 points)** A thirteen-foot-long ladder is leaning against a wall. The bottom of the ladder, which is currently five feet away from the wall, is slipping away from the wall at a rate of two feet per hour.

   (a) **(6 points)** How quickly is the top of the ladder sliding down the wall?

   (b) **(6 points)** How quickly is the angle between the ladder and the ground changing?

6. **(12 points)** Consider the function $g(x) = \frac{(x^2+6x+9}{x-2}$.

   (a) **(5 points)** Identify zeroes, vertical asymptotes, and long-term behavior on both sides of this function. Clearly label which is which.

   (b) **(5 points)** Identify the critical points of this function, and indicate whether each is a local maximum, local minimum, or neither.

   (c) **(2 points)** Which if any of the critical points identified above are global maxima or global minima? Show work or explain.

7. **(12 points)** An alien spacecraft is heated to 1700°F by entry into the atmosphere and crash-lands on an 100°F day during the summer. After 20 minutes, the ship's hull has cooled to 1300°F.

   (a) **(4 points)** Construct a function modeling the temperature of the spacecraft t minutes after impact.

   (b) **(4 points)** The scientific survey team can begin their experiments as soon as the ship has cooled to 500°F. How long will they need to wait after impact to do so?

   (c) **(4 points)** How quickly is the spacecraft cooling 10 minutes after impact?

8. **(12 points)** The equation $(x-1)(x^2+y^2) = 5x^2$ describes a figure known as the *conchoid of de Sluze*.

   (a) **(9 points)** Find a formula on the conchoid for $\frac{dy}{dx}$ in terms of x and y.

   (b) **(3 points)** Find the equation of the tangent line to the conchoid at $(2, 4)$.

9. **(12 points)** Answer the following questions:

   (a) **(4 points)** Find $\frac{d}{dx}\cos\left(e^{\sec x}\right)$.

   (b) **(4 points)** Given $g(r) = \frac{d}{dr}\arcsin(r\ln r)$, find $g'(r)$.

   (c) **(4 points)** Find $\frac{d}{d\theta}\int 3\ln|\tan\theta|d\theta$.

10. **(12 points)** Determine the following limits.

(a) **(4 points)** *Using the difference quotient*, find the derivative with respect to x of $f(x) = 2x^2 - 3x + 1$. You may not use L'Hôpital's rule for this problem.

(b) **(4 points)** Evaluate $\lim_{\theta \to 0} \frac{\theta \sin \theta}{\theta^3 - \theta^2}$ or demonstrate that it cannot be evaluated.

(c) **(4 points)** Evaluate $\lim_{x \to \infty} \frac{x^2}{x \ln x}$ or demonstrate that it cannot be evaluated.