1. **(6 points)** *Predator-prey interactions are subject to periodic behaviors and can be modeled by* trigonometric functions*. In a town infested by zombies, the human population is observed to oscillate over a period of ten years. Currently, the human population is at its peak value of* 60000*; at the lowest point of the cycle the population is* 40000*. Construct a function $f(t)$ to model the human population t years from now.*

   The sinusoidal function described here has an amplitude of 10000, a vertical shift of 50000, and a period of 10. To get the peak to occur at zero, we might start with a cosine function, or apply a phase shift to the sine. We thus take a standard cosine function (amplitude 1, period 2π, vertical shift 0), and enact upon it a vertical-stretch of 10000, a vertical shift of 50000, and a horizontal stretch of $\frac{10}{2\pi}$ to get the desiired properties. The result of these actions is the function $f(t) = 10000\cos\left(\frac{2\pi}{10}t\right) + 50000$.

2. **(6 points)** *Identify the domains of the following functions:*

   (a) **(3 points)** $g(x) = \sqrt{x+4} - 3$.
   The obvious impediment to the existence of a solution to this function is when $x+4$ is negtive, since square roots of negative numbers are specifically not doable in the real numbers. So this function exists only when $x + 4 \geq 0$. Algebraic manipulation of this expression gives $x \geq -4$; this can be alternatively expressed as the interval notation $[-4, \infty)$.

   (b) **(3 points)** $f(x) = \frac{x^2-1}{(2x+3)(x-1)}$.
   Here, the only clear impediment to the existence of this function is the division: we must be careful to make sure the denominator is nonzero. Thus, we require that $(2x + 3)(x - 1) \neq 0$, which algebraically simplifies to $2x \neq -3$ and $x \neq 1$, so it is necessary that $x \neq \frac{-3}{2}$ and $x \neq 1$, or, in interval notation, $(-\infty, \frac{-3}{2}) \cup (\frac{-3}{2}, 1) \cup (1, \infty)$.

3. **(8 points)** *The drug hypercortisone-D has a half-life of 3 hours (so, after 3 hours, half the quantity in a user's system has been eliminated). Quentin Quire has just taken a 200mg dose.*

   (a) **(4 points)** *Construct a function $f(t)$ to describe the quantity of the drug in his system after t hours.*
   We want $f(0)$ to be 200, and $f(3)$ to be $200 \cdot \frac{1}{2} = 100$, and $f(6)$ to be $200 \cdot \frac{1}{4} = 50$, and so forth; since this is a decay function, an exponential model is appropriate, and we see from these examples (or familiarity with half-life formulae) that an appropriate exponential model is $f(t) = 200 \cdot \left(\frac{1}{2}\right)^{\frac{t}{3}}$.

   (b) **(4 points)** *Use your function to determine when only 5mg of the drug remain. Your answer should be in the simplest calculatable form.*

We want to solve for the value of t when $f(t) = 200$:

$$\begin{aligned}
200 \cdot (frac12)^{\frac{t}{3}} &= 5 \\
(frac12)^{\frac{t}{3}} &= \frac{1}{40} \\
\frac{t}{3} &= \log_{1/2} \frac{1}{40} \\
t &= 3 \log_{1/2}] frac140 = 3 \frac{\ln \frac{1}{40}}{\ln \frac{1}{2}} \\
t &= 3 \frac{\ln 40}{\ln 2} = 15.97 \text{ hours}
\end{aligned}$$

The last line above is cosmetic detail, and need not be included.