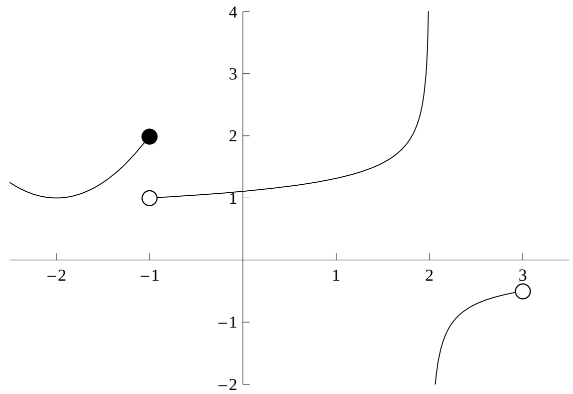
1. **(4 points)** *For the plot of $h(x)$ shown below, indicate whether or not each of the following quantities can be evaluated. If they can be evaluated, compute their values. If they cannot be evaluated, explain why.*



$\lim_{x\to -1^+} h(x) = 1$, since slightly to the right of the open circle at $(-1, 1)$, $h(x)$ is close to 1.

$\lim_{x\to 2^-} h(x)$ does not exist, and specifically increases without bound; note that slightly to the left of $x = 2$, the function is very large.

$h(-1) = 2$; note the solid dot at $(-1, 2)$.

$\lim_{x\to -2} h(x) = 1$: the function is well-behaved in the environs of $(-2, 1)$, so the function is continuous here, with this limit equal to the function evaluation $h(-2)$.

2. **(9 points)** *Evaluate the following limits, or explicitly state that they do not exist and explain why they cannot be evaluated.*

   (a) **(3 points)** $\lim_{s\to 2} \sqrt{2-s}$.

   For s slightly larger than 2, $2 - s$ is negative so $\sqrt{2-s}$ is not evaluatable in the real numbers. Thus, this limit does not exist since the right-side limit, in particular, does not exist.

   (b) **(3 points)** $\lim_{x\to 1^+} \frac{x^2-2x}{x^3+x^2+1}$.

   1 is in the domain of this rational function, so we simply evaluate it at 1 to get $\frac{1^2-2}{1^3+\cdot 1^2+1} = \frac{-1}{3}$.

   (c) **(3 points)** $\lim_{u\to 3} \frac{u-3}{u^2-4u+3}$.

   Note that $\frac{u-3}{u^2-4u+3} = \frac{u-3}{(u-3)(u-1)}$, so except at the point $u = 3$, this expression can be simplified to $\frac{1}{u-1}$. Since we are specifically *not* looking at $u = 3$, but merely at values close to 3, such a simplification is justified, and thus $\lim_{u\to 3} \frac{u-3}{u^2-4u+3} = \lim_{u\to 3} \frac{1}{u-1} = \frac{1}{2}$

3. **(3 points)** *Write the formal definition of the statement "$\lim_{x\to a^+} f(x) = -\infty$".*

For every E, there is a δ, such that if $a < x < a + \delta$, then $f(x) > E$.

4. **(4 points)** *Using epsilon-delta methods, prove that* $\lim_{x \to 4} -4x + 2 = -14$.

Given an ϵ, we wish to establish a range on values of x such that $-4x + 2$ is within ϵ of -14, that is to say, between $-14 - \epsilon$ and $-14 + \epsilon$. We algebraically manipulate this desideratum:

$$\begin{aligned} -14 - \epsilon < -4x + 2 < -14 + \epsilon \\ -16 - \epsilon < -4x < -16 + \epsilon \\ \frac{-16 - \epsilon}{-4} > x > \frac{-16 + \epsilon}{-4} \\ 4 + \frac{\epsilon}{4} > x > 4 - \frac{\epsilon}{4} \end{aligned}$$

So in order to get $-4x + 2$ within ϵ of -14, we need to get x within $\frac{\epsilon}{4}$ of 4. Thus, $\delta = \frac{\epsilon}{4}$ suffices.