1. **(4 points)** *Given $f(x) = 6x + 1$, use the difference quotient to calculate $f'(x)$.*

$$\begin{aligned} f'(x) &= \lim_{h\to 0} \frac{f(x+h) - f(x)}{h} \\ &= \lim_{h\to 0} \frac{[6(x+h)+1] - (6x+1)}{h} \\ &= \lim_{h\to 0} \frac{6x + 6h + 1 - 6x - 1}{h} \\ &= \lim_{h\to 0} \frac{6h}{h} \\ &= \lim_{h\to 0} 6 \text{ justified since } h \neq 0 = 6 \end{aligned}$$

2. **(4 points)** *Determine the values of the following limits if possible; if they can not be evaluated, explain why.*

   (a) **(2 points)** $\lim_{u\to+\infty} \frac{7u^2-3}{u^4+u}$.
   Considering only the highest order terms, we can see that

$$\lim_{u\to+\infty} \frac{7u^2-3}{u^4+u} = \lim_{u\to+\infty} \frac{7u^2}{u^4} = \lim_{x\to+\infty} \frac{7}{u^2} = 0$$

   If considering only the highest order terms does not appeal, one can instead divide the numerator and denominator by u^4 and reduce the limit using the known fact that $\lim_{u\to\pm\infty} \frac{1}{u^k} = 0$ for positive k:

$$\lim_{u\to+\infty} \frac{7u^2-3}{u^4+u} = \lim_{u\to+\infty} \frac{\frac{7}{u^2} - \frac{3}{u^4}}{1 + \frac{1}{u^3}} = \frac{0-0}{1+0} = 0$$

   (b) **(2 points)** $\lim_{t\to-\infty} \frac{t^6+3t^2-100}{2t^6+t^3}$.
   Considering only the highest order terms, we can see that

$$\lim_{t\to-\infty} \frac{t^6+3t^2-100}{2t^6+t^3} = \lim_{t\to-\infty} \frac{t^6}{2t^6} = \frac{1}{2}$$

   If considering only the highest order terms does not appeal, one can instead divide the numerator and denominator by t^6 and reduce the limit using the known fact that $\lim_{t\to\pm\infty} \frac{1}{t^k} = 0$ for positive k:

$$\lim_{t\to-\infty} \frac{t^6+3t^2-100}{2t^6+t^3} = \lim_{t\to-\infty} \frac{1 + \frac{3}{t^4} - \frac{100}{t^6}}{2 + \frac{1}{t^3}} = \frac{1+0-0}{2+0} = \frac{1}{2}$$

3. **(4 points)** *Calculate $\frac{d}{dx}\left(\frac{7}{x^2} - 5 + 3\sqrt{x}\right)$.*
   Rephrasing radicals and division, this expression is simply $\frac{d}{dx}\left(7x^{-2} - 5 + 3x^{1/2}\right)$, which is amenable to power rule evaluation to become $-14x^{-3} + \frac{3}{2}x^{-\frac{1}{2}} = \frac{-14}{x^3} + \frac{3}{2\sqrt{x}}$.

   It is possible to differentiate $\frac{7}{x^2}$ using the quotient rule, but doing so is generally regarded as more difficult than using the power rule directly.

4. **(8 points)** *Solve the problems given below.*

   (a) **(4 points)** *Find* $\frac{d}{dt}\left(t^3 e^t\right)$.

   We can use the product rule:

$$\begin{aligned}\frac{d}{dt}\left(t^3 e^t\right) &= \left(\frac{d}{dt}t^3\right)e^t + t^3\frac{d}{dt}e^t \\ &= 3t^2e^t + t^3e^t\end{aligned}$$

   (b) **(4 points)** Given $g(x) = \frac{x^2-3x}{2x^3-1}$, find $g'(x)$.

   $g'(x) = \frac{d}{dx}\frac{x^2-3x}{2x^3-1}$, which we can evaluate using the quotient rule:

$$\begin{aligned}g'(x) &= \frac{d}{dx}\frac{x^2-3x}{2x^3-1} \\ &= \frac{(2x^3-1)\frac{d}{dx}(x^2-3x) - (x^2-3x)\frac{d}{dx}(2x^3-1)}{(2x^3-1)^2} \\ &= \frac{(2x^3-1)(2x-3) - (x^2-3x)(6x^2)}{(2x^3-1)^2}\end{aligned}$$