1. **(6 points)** *Answer the following questions about the ellipse defined by the equation* $2x^2 + 3y^2 - 4xy + 6y = -3$.

   (a) **(4 points)** *On this curve, find* $\frac{dy}{dx}$ *in terms of* x *and* y.
   We implicitly differentiate the equation, using the chain and product rules as appropriate, then gather terms:

$$\begin{aligned}
\frac{d}{dx}(2x^2 + 3y^2 - 4xy + 6y) &= \frac{d}{dx}(-3) \\
4x + 3\frac{d}{dx}y^2 - 4\frac{d}{dx}(xy) + 6\frac{dy}{dx} &= 0 \\
4x + 3\frac{dy}{dx}\frac{d}{dy}y^2 - 4\left(\frac{d}{dx}x\right)y + x\left(\frac{d}{dx}y\right) + 6\frac{dy}{dx} &= 0 \\
4x + 6\frac{dy}{dx}y - 4y - 4x\frac{dy}{dx} + 6\frac{dy}{dx} &= 0 \\
6\frac{dy}{dx}y - 4x\frac{dy}{dx} + 6\frac{dy}{dx} &= 4y - 4x \\
(6y - 4x + 6)\frac{dy}{dx} &= 4y - 4x \\
\frac{dy}{dx} &= \frac{4y - 4x}{6y - 4x + 6}
\end{aligned}$$

   (b) **(2 points)** *Find the equation of the tangent line to this curve at* $(-2, -1)$.
   When $x = -2$ and $y = -1$, we know from the above formula for $\frac{dy}{dx}$ that

$$\frac{dy}{dx} = \frac{4(-1) - 4(-2)}{6(-1) - 4(-2) + 6} = \frac{4}{8} = \frac{1}{2}$$

   so the tangent line to the curve at $(1, -2)$ has slope $\frac{1}{2}$, and thus has equation, in point-slope form, of

$$(y + 1) = \frac{1}{2}(x + 2)$$

   or, in slope-intercept form, $y = \frac{1}{2}x$.

2. **(14 points)** Answer the questions below:

   (a) **(4 points)** *If* $w = \sec(\sqrt{x})$, *find* $\frac{dw}{dx}$.
   This is a chain-rule application, since a simple function (the secant) is being applied to a complex expression ($\sqrt{x}$). We give the complex expression a name, defining $u = \sqrt{x}$, and then invoke the chain rule:

$$\begin{aligned}
\frac{dw}{dx} &= \frac{d}{dx}\sec u \\
&= \frac{du}{dx}\frac{d}{du}\sec u \\
&= \left(\frac{d}{dx}x^{1/2}\right)\left(\frac{d}{du}\sec u\right) \\
&= \left(\frac{1}{2}x^{-1/2}\right)(\sec u \tan u) \\
&= \left(\frac{1}{2}x^{-1/2}\right)(\sec\sqrt{u}\tan\sqrt{u}) = \frac{\sec\sqrt{u}\tan\sqrt{u}}{2\sqrt{u}}
\end{aligned}$$

   The very last equality is a purely cosmetic simplification and is not necessary.

(b) **(5 points)** *Find* $\frac{d}{dt}\cot\frac{t^2}{t-2}$.
We can see that this is a chain rule problem, since a manageable function, the cotangent, is being applied to a complicated expression, which is $\frac{t^2}{t-2}$. We further anticipate using the quotient rule to differentiate this expression. letting $u = \frac{t^2}{t-2}$, we invoke the chain and quotient rules in turn:

$$\begin{aligned}
\frac{d}{dt}\cot\frac{t^2}{t-2} &= \frac{d}{dt}\cot u \\
&= \frac{du}{dt}\frac{d}{du}\cot u \\
&= \left(\frac{d}{dt}\frac{t^2}{t-2}\right)\left(\frac{d}{du}\cot u\right) \\
&= \left(\frac{(t-2)\frac{d}{dt}(t^2) - t^2\frac{d}{dt}(t-2)}{(t-2)^2}\right)\left(-\csc^2 u\right) \\
&= \left(\frac{(t-2)(2t) - t^2(1)}{(t-2)^2}\right)\left(-\csc^2\frac{t^2}{t-2}\right)
\end{aligned}$$

(c) **(5 points)** *If* $g(x) = e^{x\arctan x}$, *find* $g'(x)$.
We can see that this is a chain rule problem, since a manageable function, the exponential, is being applied to a complicated expression, which is $x\arctan x$. We further anticipate using the product rule to differentiate this expression. letting $u = x\arctan x$, we invoke the chain and product rules in turn:

$$\begin{aligned}
g'(x) &= \frac{d}{dx}e^u \\
&= \frac{du}{dx}\frac{d}{du}e^u \\
&= \left(\frac{d}{dx}(x\arctan x)\right)\left(\frac{d}{du}e^u\right) \\
&= \left[\left(\frac{d}{dx}x\right)\arctan x + x\left(\frac{d}{dx}\arctan x\right)\right](e^u) \\
&= \left[\arctan x + x\cdot\frac{1}{1+x^2}\right]\left(e^{x\arctan x}\right)
\end{aligned}$$