Show work for each problem. Answers without justification, or justified solely by direct enumeration, are not acceptable. Arithmetic expressions may be left unsimplified (e.g. if you can explain why an answer is computable as being $6^7-5\cdot 2^3$, that is more than sufficient; it is not necessary to then calculate that $6^7-5\cdot 2^3$ is 279896).

This problem set is due at the beginning of class on *September 3*.

1. **(10 points)** A five-digit number must not have a zero as its first digit.
   (a) **(5 points)** How many five-digit numbers are there whose digits appear in increasing order (e.g., 25689)?
   (b) **(5 points)** How many five-digit numbers are there whose digits appear in descending order (e.g., 98652)?

2. **(10 points)** The members of two committees are being drawn from a pool of eight people. Each person can be on either committee or on no committee at all.
   (a) **(5 points)** How many ways are there to assign committee-responsibilities to the eight people if there are no restrictions on committee membership?
   (b) **(5 points)** How many ways are there to assign committee-responsibilities to the eight people if each committee must have at least one member?

3. **(10 points)** There are two straightforward ways to prove that $\binom{2n}{2} = 2\binom{n}{2} + n^2$.
   (a) **(5 points)** Prove it algebraically, by manipulating the two sides to get equal expressions.
   (b) **(5 points)** Prove it combinatorially, by finding a set counted by the left side and demonstrating that the right side counts the same set.

4. **(10 points)** Prove the following identity by demonstrating that the two sides of the equation are counting the same thing:

$$\sum_{k=0}^{n} \binom{n}{k}\binom{n}{n-k} = \binom{2n}{n}$$

Brute force is the last refuge of the incompetent. —*Traditional hacker lore*