1. (10 points) Let \( f(x) = 3x^2 - 6 \).

   (a) (6 points) Using the difference quotient, find \( f'(x) \).

\[
f'(x) = \lim_{h \to 0} \frac{f(x + h) - f(x)}{h}
= \lim_{h \to 0} \frac{[3(x + h)^2 - 6] - (3x^2 - 6)}{h}
= \lim_{h \to 0} \frac{3x^2 + 6xh + 3h^2 - 6 - 3x^2 + 6}{h}
= \lim_{h \to 0} \frac{6xh + 3h^2}{h}
= \lim_{h \to 0} 6x + 3h \text{ justified since } h \neq 0
= 6x
\]

(b) (4 points) Find the equation of the tangent line to \( f(x) \) at the point \((2, 6)\).

We know this line must pass through \((2, 6)\), with slope \( f'(2) = 6 \cdot 2 = 12 \). The equation of the line is thus known to be \( y = 12x + b \), and putting in the known values \((2, 6)\) will give us \( b: 6 = 12 \cdot 2 + b \), which leads to the fact that \( b = -18 \). Thus the equation of this line is \( y = 12x - 18 \).

2. (10 points) For the plot of \( f(x) \) shown below, indicate whether or not each of the following quantities can be evaluated. If they can be evaluated, compute their values. If they cannot be evaluated, explain why.

\[
\begin{array}{c}
\lim_{x \to -1^-} f(x) = 1, \text{ since slightly to the left of the solid dot at } (-1, 1), \text{ } f(x) \text{ is close to 1.} \\
f(2) = -2, \text{ as indicated by the solid dot.} \\
\lim_{x \to -2^-} f(x) \text{ does not exist, since } f(x) \text{ decreases without bound as } x \text{ approaches } -2 \text{ from below.} \\
\lim_{x \to 2^+} f(x) = 2, \text{ since at } x\text{-values close to } 2, \text{ } f(x) \text{ takes on values close to } 2 \text{ (as indicated by the curves near the open circle at } (2, 2)\text{).}
\end{array}
\]

3. (10 points) Find the derivatives of the following functions:

   (a) (4 points) \( f(x) = 5x - 3\sqrt{x} \).
If we rephrase this as \( f(x) = 5x + 3\sqrt{x} \), then we may use the power rule, together with the additive and scalar-constant multiplication properties of the derivative, to find that \( f'(x) = 5(1x^0) - 3 \left( \frac{1}{2}x^{-1/2} \right) = 5 - \frac{3}{2}x^{-1/2} \).

(b) (3 points) \( g(x) = x^2 - \frac{1}{x} + \frac{3}{x^2} \).

If we rephrase this as \( f(x) = x^2 - x^{-1} + 3x^{-4} \), then we may use the power rule, together with the additive and scalar-constant multiplication properties of the derivative, to find that \( f'(x) = 2x^1 - (-1x^{-2}) + 3(-4x^{-5}) = 2x + x^{-2} - 12x^{-5} \).

(c) (3 points) \( h(x) = x^5 - 3x^2 + 2x - 1 \).

Using the power rule, together with the additive and scalar-constant multiplication properties of the derivative, we find that \( f'(x) = 5x^4 - 3(2x^1) + 2(1x^0) - (0x^{-1}) = 5x^4 - 6x + 2 \).

4. (10 points) Determine the domains of the following functions:

(a) (5 points) \( f(x) = \frac{\sqrt{x+5}}{x-3} \).

Division by zero occurs when \( x - 3 = 0 \); in addition, the square-root of a negative number is taken when \( x + 5 < 0 \). Thus, the domain consists of those values where \( x \neq 3 \) and \( x \geq -5 \); alternatively, in interval notation, \( (-5, 3) \cup (3, \infty) \).

(b) (5 points) \( g(t) = \frac{3t-6}{(t+1)(t-4)} \).

Division by zero occurs when \( t + 1 = 0 \) or \( t - 4 = 0 \); thus, the domain must exclude the \( t \)-values of -1 and 4. Thus the domain is those values where \( t \neq -1 \) and \( t \neq 4 \); or alternatively in interval notation, \( (-\infty, -1) \cup (-1, 4) \cup (4, \infty) \).

5. (15 points) Evaluate the following limits; when a limit cannot be evaluated, explain why.

(a) (3 points) \( \lim_{r \to 2} \frac{r^2 + 2r + 1}{r - 3} \).

This rational function is defined at \( r = 2 \), so its limit is simply the evaluation \( \frac{2^2 + 2 \cdot 2 + 1}{2 - 3} = \frac{9}{-1} = -9 \).

(b) (3 points) \( \lim_{x \to \infty} \frac{2x^2 - 3}{x^3 - 2x} \).

Dividing by the highest-degree term appearing in the rational expression, which is \( x^3 \), we get that \( \lim_{x \to +\infty} \frac{2x^2 - 3}{5x^3 - 2x} = \lim_{x \to +\infty} \frac{2}{5} \frac{x^2 - \frac{3}{x^2}}{x^3 - \frac{2}{x}} = \frac{2}{5} \frac{0 - 0}{0} = 0 \).

(c) (3 points) \( \lim_{t \to -3} \frac{t^2 + 5x + 6}{t + 3} \).

Since we look near, but not at, \( t = -3 \), we can justify the cancellation \( \lim_{t \to -3} \frac{t^2 + 5x + 6}{t + 3} = \lim_{t \to -3} \frac{(t+2)(t+3)}{t+3} = \lim_{t \to -3} t + 2 = -3 + 2 = -1 \).

(d) (3 points) \( \lim_{u \to 4} u^2 - 7u + 12 \).

This function is a polynomial, so a limit of this function as \( x \) approaches a value is identical to a direct evaluation: thus, \( \lim_{u \to 4} u^2 - 7u + 12 = 4^2 - 7 \cdot 4 + 12 = 0 \).

(e) (3 points) \( \lim_{x \to -\infty} \frac{x^2 - 3x + 2}{4x^2 + 4x + 1} \).
Dividing by the highest-degree term appearing in the rational expression, which is $x^2$, we get that

$$
\lim_{x \to -\infty} \frac{x^2 - 3x + 2}{4x^2 + 4x + 1} = \lim_{x \to -\infty} \frac{1 - \frac{3}{x} + \frac{2}{x^2}}{4 + \frac{4}{x} + \frac{1}{x^2}} = \frac{1 - 0 + 0}{4 + 0 + 0} = \frac{1}{4}
$$

6. **(5 point bonus)** Use the difference quotient to find the derivative of the function $f(x) = \frac{1}{x^2}$.

$$
f'(x) = \lim_{h \to 0} \frac{f(x + h) - f(x)}{h}
$$

$$
= \lim_{h \to 0} \frac{\frac{1}{(x+h)^2} - \frac{1}{x^2}}{h}
$$

$$
= \lim_{h \to 0} \frac{x^2 - (x+h)^2}{h \cdot x^2(x+h)^2}
$$

$$
= \lim_{h \to 0} \frac{x^2 - (x^2 + 2xh + h^2)}{x^2(x+h)^2h}
$$

$$
= \lim_{h \to 0} \frac{-2xh - h^2}{x^2(x+h)^2h}
$$

$$
= \lim_{h \to 0} \frac{-2x - h}{x^2(x+h)^2}
$$

$$
= \frac{-2x - 0}{x^2(x+0)^2} = \frac{-2x}{x^4} = -\frac{2}{x^3}
$$