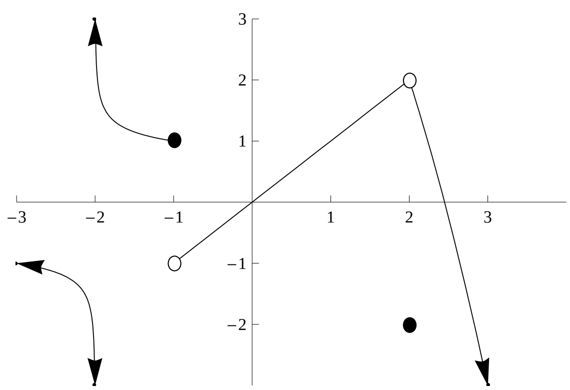
1. **(10 points)** *Let* $f(x) = 3x^2 - 6$.

   (a) **(6 points)** *Using the difference quotient, find* $f'(x)$.

$$\begin{aligned} f'(x) &= \lim_{h\to 0} \frac{f(x+h) - f(x)}{h} \\ &= \lim_{h\to 0} \frac{[3(x+h)^2 - 6] - (3x^2 - 6)}{h} \\ &= \lim_{h\to 0} \frac{3x^2 + 6xh + 3h^2 - 6 - (3x^2 - 6)}{h} \\ &= \lim_{h\to 0} \frac{6xh + 3h^2}{h} \\ &= \lim_{h\to 0} 6x + 3h \text{ justified since } h \neq 0 \\ &= 6x \end{aligned}$$

   (b) **(4 points)** *Find the equation of the tangent line to* $f(x)$ *at the point* $(2, 6)$.
   We know this line must pass through $(2, 6)$, with slope $f'(2) = 6 \cdot 2 = 12$. The equation of the line is thus known to be $y = 12x + b$, and putting in the known values $(2, 6)$ will give us b: $6 = 12 \cdot 2 + b$, which leads to the fact that $b = -18$. Thus the equation of this line is $y = 12x - 18$.

2. **(10 points)** *For the plot of* $f(x)$ *shown below, indicate whether or not each of the following quantities can be evaluated. If they can be evaluated, compute their values. If they cannot be evaluated, explain why.*



$\lim_{x\to -1^-} f(x) = 1$, since slightly to the left of the solid dot at $(-1, 1)$, $f(x)$ is close to 1.

$f(2) = -2$, as indicated by the solid dot.

$\lim_{x\to -2^-} f(x)$ does not exist, since $f(x)$ decreases without bound as x approaches -2 from below.

$\lim_{x\to 2} f(x) = 2$, since at x-values close to 2, $f(x)$ takes on values close to 2 (as indicated by the curves near the open circle at $(2, 2)$.

3. **(10 points)** *Find the derivatives of the following functions:*

   (a) **(4 points)** $f(x) = 5x - 3\sqrt{x}$.

If we rephrase this as $f(x) = 5x + 3\sqrt{x}$, then we may use the power rule, together with the additive and scalar-constant multiplication properties of the derivative, to find that $f'(x) = 5\left(1x^0\right) - 3\left(\frac{1}{2}x^{-1/2}\right) = 5 - \frac{3}{2}x^{-1/2}$.

(b) **(3 points)** $g(x) = x^2 - \frac{1}{x} + \frac{3}{x^4}$.

If we rephrase this as $f(x) = x^2 - x^{-1} + 3x^{-4}$, then we may use the power rule, together with the additive and scalar-constant multiplication properties of the derivative, to find that $f'(x) = 2x^1 - (-1x^{-2}) + 3(-4x^{-5}) = 2x + x^{-2} - 12x^{-5}$.

(c) **(3 points)** $h(x) = x^5 - 3x^2 + 2x - 1$.

Using the power rule, together with the additive and scalar-constant multiplication properties of the derivative, we find that $f'(x) = 5x^4 - 3(2x^1) + 2(1x^0) - (0x^{-1}) = 5x^4 - 6x + 2$.

4. **(10 points)** *Determine the domains of the following functions:*

(a) **(5 points)** $f(x) = \frac{\sqrt{x+5}}{x-3}$.

Division by zero occurs when $x - 3 = 0$; in addition, the square-root of a negative number is taken when $x + 5 < 0$. Thus, the domain consists of those values where $x \neq 3$ and $x \geq -5$; alternatively, in interval notation, $(-5, 3) \cup (3, \infty)$.

(b) **(5 points)** $g(t) = \frac{3t-6}{(t+1)(t-4)}$.

Division by zero occurs when $t + 1 = 0$ or $t - 4 = 0$; thus, the domain must exclude the t-values of -1 and 4. Thus the domain is those values where $t \neq -1$ and $t \neq 4$; or alternatively in interval notation, $(\infty, -1) \cup (-1, 4) \cup (4, \infty)$.

5. **(15 points)** *Evaluate the following limits; when a limit can not be evaluated, explain why.*

(a) **(3 points)** $\lim\limits_{r\to 2} \frac{r^2+2r+1}{r-3}$.

This rational function is defined at $r = 2$, so its limit is simply the evaluation $\frac{2^2+2\cdot 2+1}{2-3} = \frac{9}{-1} = -9$.

(b) **(3 points)** $\lim\limits_{x\to\infty} \frac{2x^2-3}{5x^3-2x}$.

Dividing by the highest-degree term appearing in the rational expression, which is x^3, we get that

$$\lim_{x\to+\infty} \frac{2x^2-3}{5x^3-2x} = \lim_{x\to+\infty} \frac{\frac{2}{x} - \frac{3}{x^3}}{5 - \frac{2}{x^2}} = \frac{0-0}{5-0} = \frac{0}{5} = 0$$

(c) **(3 points)** $\lim\limits_{t\to -3} \frac{t^2+5x+6}{t+3}$.

Since we look near, but not at, $t = -3$, we can justify the cancellation $\lim_{t\to-3} \frac{t^2+5x+6}{t+3} = \lim_{t\to-3} \frac{(t+2)(t+3)}{t+3} = \lim_{t\to-3} t + 2 = -3 + 2 = -1$.

(d) **(3 points)** $\lim\limits_{u\to 4} u^2 - 7u + 12$.

This function is a polynomial, so a limit of this function as x approaches a value is identical to a direct evaluation: thus, $\lim\limits_{u\to 4} u^2 - 7u + 12 = 4^2 - 7\cdot 4 + 12 = 0$.

(e) **(3 points)** $\lim\limits_{x\to-\infty} \frac{x^2-3x+2}{4x^2+4x+1}$.

Dividing by the highest-degree term appearing in the rational expression, which is x^2, we get that

$$\lim_{x\to-\infty}\frac{x^2-3x+2}{4x^2+4x+1}=\lim_{x\to-\infty}\frac{1-\frac{3}{x}+\frac{2}{x^2}}{4+\frac{4}{x}+\frac{1}{x^2}}=\frac{1-0+0}{4+0+0}=\frac{1}{4}$$

6. **(5 point bonus)** *Use the difference quotient to find the derivative of the function* $f(x)=\frac{1}{x^2}$.

$$\begin{aligned}
f'(x)&=\lim_{h\to0}\frac{f(x+h)-f(x)}{h}\\
&=\lim_{h\to0}\frac{\frac{1}{(x+h)^2}-\frac{1}{x^2}}{h}\\
&=\lim_{h\to0}\frac{\frac{x^2}{x^2(x+h)^2}-\frac{(x+h)^2}{x^2(x+h)^2}}{h}\\
&=\lim_{h\to0}\frac{x^2-(x+h)^2}{x^2(x+h)^2h}\\
&=\lim_{h\to0}\frac{x^2-(x^2+2xh+h^2)}{x^2(x+h)^2h}\\
&=\lim_{h\to0}\frac{-2xh-h^2}{x^2(x+h)^2h}\\
&=\lim_{h\to0}\frac{-2x-h}{x^2(x+h)^2}\\
&=\frac{-2x-0}{x^2(x+0)^2}=\frac{-2x}{x^4}=\frac{-2}{x^3}
\end{aligned}$$