

1. (10 points) Let $f(x) = 2x^2 - 3x$.

(a) (6 points) Using the difference quotient, find $f'(x)$.

$$\begin{aligned} f'(x) &= \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \\ &= \lim_{h \rightarrow 0} \frac{[2(x+h)^2 - 3(x+h)] - (2x^2 - 3x)}{h} \\ &= \lim_{h \rightarrow 0} \frac{2x^2 + 4xh + 2h^2 - 3x - 3h - (2x^2 - 3x)}{h} \\ &= \lim_{h \rightarrow 0} \frac{4xh + 2h^2 - 3h}{h} \\ &= \lim_{h \rightarrow 0} 4x + 2h - 3 \text{ justified since } h \neq 0 \\ &= 4x - 3 \end{aligned}$$

(b) (4 points) Find the equation of the tangent line to $f(x)$ at the point $(-1, 5)$.

We know this line must pass through $(-1, 5)$, with slope $f'(-1) = 4(-1) - 3 = -7$. The equation of the line is thus known to be $y = -7x + b$, and putting in the known values $(-1, 5)$ will give us b : $5 = -7(-1) + b$, which leads to the fact that $b = -2$. Thus the equation of this line is $y = -7x - 2$.

2. (15 points) Evaluate the following limits; when a limit can not be evaluated, explain why.

(a) (3 points) $\lim_{u \rightarrow 1} \frac{(u^2 - 4u + 3)}{(u-1)}$.

Since we look near, but not at, $u = 1$, we can justify the cancellation $\lim_{u \rightarrow 1} \frac{(u^2 - 4u + 3)}{(u-1)} = \lim_{u \rightarrow 1} \frac{(u-3)(u-1)}{(u-1)} = \lim_{u \rightarrow 1} u - 3 = -2$.

(b) (3 points) $\lim_{x \rightarrow +\infty} \frac{5x^3 - 2x + 1}{4x^3 + 2x^2}$.

Dividing by the highest-degree term appearing in the rational expression, which is x^3 , we get that

$$\lim_{x \rightarrow +\infty} \frac{5x^3 - 2x + 1}{4x^3 + 2x^2} = \lim_{x \rightarrow +\infty} \frac{5 - \frac{2}{x^2} + \frac{1}{x^3}}{4 + \frac{2}{x}} = \frac{5 - 0 + 0}{4 + 0} = \frac{5}{4}$$

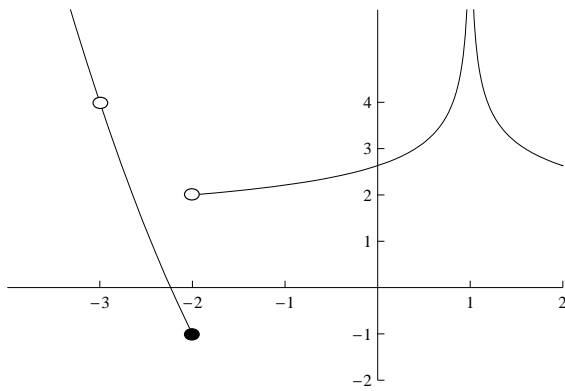
(c) (3 points) $\lim_{x \rightarrow 5} x^2 + 2x - 3$.

This function is a polynomial, so a limit of this function as x approaches a value is identical to a direct evaluation: thus, $\lim_{x \rightarrow 5} x^2 + 2x - 3 = 5^2 + 2 \cdot 5 - 3 = 32$.

(d) (3 points) $\lim_{t \rightarrow 2} \frac{t^2 - 4}{t + 3}$.

This rational function is defined at $t = 2$, so its limit is simply the evaluation $\frac{2^2 - 4}{2 + 3} = \frac{0}{5} = 0$.

3. (10 points) For the plot of $g(x)$ shown below, indicate whether or not each of the following quantities can be evaluated. If they can be evaluated, compute their values. If they cannot be evaluated, explain why.



$g(-2) = -1$, as indicated by the solid dot.

$\lim_{x \rightarrow -3^-} g(x) = 4$, since slightly to the left of the open circle at $(-3, 4)$, $f(x)$ is close to 4.

$\lim_{x \rightarrow -2} g(x)$ does not exist because of a jump discontinuity.

$\lim_{x \rightarrow 1^+} g(x)$ does not exist, since $g(x)$ increases without bound as x approaches 1 from above.

4. **(10 points)** For the plot of $g(x)$ shown below, indicate whether or not each of the following quantities can be evaluated. If they can be evaluated, compute their values. If they cannot be evaluated, explain why.

5. **(10 points)** Determine the domains of the following functions:

(a) **(3 points)** $f(x) = x^2 - 3x + 2$.

This is a polynomial, so every real number is in its domain; this could be expressed in words, or in interval notation as $(-\infty, \infty)$.

(b) **(3 points)** $g(t) = \frac{1}{t-2} + \frac{1}{t}$.

Division by zero occurs when $t - 2 = 0$ or $t = 0$; thus, the domain must exclude the t -values of 0 and 2. Thus the domain is those values where $t \neq 0$ and $t \neq 2$; or alternatively in interval notation, $(-\infty, 0) \cup (0, 2) \cup (2, \infty)$.

(c) **(4 points)** $h(u) = \frac{\sqrt{u+2}}{u-4}$.

Division by zero occurs when $u - 4 = 0$; in addition, the square-root of a negative number is taken when $u + 2 < 0$. Thus, the domain consists of those values where $u \neq 4$ and $u \geq -2$; alternatively, in interval notation, $(-2, 4) \cup (4, \infty)$.

6. **(10 points)** Find the derivatives of the following functions:

• **(3 points)** $f(x) = 2x^{10} - 3x + 4$.

Using the power rule, together with the additive and scalar-constant multiplication properties of the derivative, we find that $f'(x) = 2(10x^9) - 3(1x^0) + 4(0x^{-1}) = 20x^9 - 3$.

• **(3 points)** $f(x) = \frac{4}{x} - \frac{2}{x^3}$.

If we rephrase this as $f(x) = 4x^{-1} - 2x^{-3}$, then we may use the power rule, together with the additive and scalar-constant multiplication properties of the derivative, to find that $f'(x) = 4(-1x^{-2}) - 2(-3x^{-4}) = -4x^{-2} + 6x^{-4} = \frac{6}{x^4} - \frac{4}{x^2}$.

- **(4 points)** $f(x) = 3\sqrt{x} + x^{5/4} - 7$.

If we rephrase this as $f(x) = 3x^{1/2} + x^{5/4} - 7$, then we may use the power rule, together with the additive and scalar-constant multiplication properties of the derivative, to find that $f'(x) = 3\left(\frac{1}{2}x^{-1/2}\right) + \frac{5}{4}x^{1/4} - 7(0x^{-1}) = \frac{3}{2}x^{-1/2} + \frac{5}{4}x^{1/4} = \frac{3}{2\sqrt{x}} + \frac{5}{4}\sqrt[4]{x}$.