1. **(3 points)** *Identify the domain of the following functions:*

   (a) **(3 points)** $f(x) = \frac{4x}{x+2}$.
   Division by zero is impossible, so it must be the case that $x + 2 \neq 0$; thus the function is defined only when $x \neq -2$. The domain could alternatively be given as $(-\infty, -2) \cup (-2, \infty)$.

   (b) **(3 points)** $g(t) = \sqrt{t-3}$.
   The square root of a negative number is problematic, so in order for this function to be evaluatable, it must be the case that $t - 3 \geq 0$, so the function is defined only when $t \geq 3$, which could alternatively be stated in interval notation as $[3, \infty)$

2. **(7 points)** *For* $f(x) = 2x^2$ *and* $g(x) = \frac{1}{3x+1}$, *determine* $f(g(x))$ *and* $g(f(x))$.

$$f(g(x)) = f\left(\frac{1}{3x+1}\right) = 2\left(\frac{1}{3x+1}\right)^2 = \frac{2}{(3x+1)^2}$$
$$g(f(x)) = g(2x^2) = \frac{1}{3(2x^2)+1} = \frac{1}{6x^2+1}$$

The last step in each of the above calculations is for simplification and aesthetic purposes and need not be included.

3. **(7 points)** *Yoyodyne Industries' daily revenue is a linear function of the number of workers they have. When they have 50 employees, they make \$40000 daily; when they have 100 employees, their daily revenue is \$50000.*

   (a) **(6 points)** *Find the daily revenue as a function* $f(x)$ *of the number of workers* x.
   We know the revenue is a linear function of the number of workers, so $f(x) = mx + b$ for some constants m and b. We also know the revenue when there are 50 and 100 workers; specifically, it follows that $f(50) = 40000$ and $f(100) = 50000$. Evaluating the linear expression for each of these, we have two equations in m and b:

$$\begin{cases} 40000 = f(\ 50) = m(50) + b \\ 50000 = f(100) = m(100) + b \end{cases}$$

   The easiest way to find m is to subtract the former equation from the latter, giving $10000 = 50m$, so $m = \frac{10000}{50} = 200$. To find b, we simply substitute our known value of m into the equation $40000 = m(50)+b$, so $40000 = 200(50)+b$ and thus $b = 40000-200\cdot 50 = 30000$. Thus the function sought is $f(x) = 200x + 30000$.

   (b) **(1 points)** *What would their daily revenue be if they employed 60 people?*
   This quantity is $f(60) = 200(60) + 30000 = 42000$.

   (c) **(2 point bonus)** *Construct a function* $g(x)$ *describing* per-worker *revenue as a function of the number of workers* x.
   We know the total revenue is $f(x) = 200x + 30000$. To determine revenue per worker, one would divide the total revenue by the number of workers, which is, by our statements throughout this problem, established to be simply x. Thus, the per-worker revenue is $g(x) = \frac{f(x)}{x} = \frac{200x+30000}{x} = 200 + \frac{30000}{x}$.