

1. **(4 points)** Find the equation of a tangent line to the curve $f(x) = x^4$ at $(2, 16)$.

The power rule informs us that $f'(x) = 4x^3$; we know this line must pass through $(2, 16)$, with slope $f'(2) = 4 \cdot 2^3 = 32$. The equation of the line is thus known to be $y = 32x + b$, and putting in the known values $(2, 16)$ will give us b : $16 = 32 \cdot 2 + b$, which leads to the fact that $b = -48$. Thus the equation of this line is $y = 32x - 48$.

2. **(4 points)** For $f(t) = (4t^3 - 3t^2 + 2t - 1)$, evaluate $f'(t)$.

Using the power rule, together with the additive and scalar-constant multiplication properties of the derivative, we find that $f'(t) = 4(3t^2) - 3(2t^1) + 2(1x^0) - (0x^{-1}) = 12t^2 - 6t + 2$.

3. **(4 points)** Evaluate $\frac{d}{dx} (4\sqrt{x} - \frac{5}{x^6})$.

If we rephrase this expression as $\frac{d}{dx} (4x^{1/2} - 5x^{-6})$, then we may use the power rule, together with the additive and scalar-constant multiplication properties of the derivative, to find that $\frac{d}{dx} (4x^{1/2} - 5x^{-6}) = 4(\frac{1}{2}x^{-1/2}) - 5(-6x^{-7}) = 2x^{-1/2} + 30x^{-7}$.

4. **(4 points)** If $f(x) = \frac{3x-2}{x^2-4x}$, determine $f'(x)$.

Using the quotient rule,

$$\begin{aligned} f'(x) &= \frac{d}{dx} \frac{3x-2}{x^2-4x} \\ &= \frac{(x^2-4x)\frac{d}{dx}(3x-2) - (3x-2)\frac{d}{dx}(x^2-4x)}{(x^2-4x)^2} \\ &= \frac{(x^2-4x)(3) - (3x-2)(2x-4)}{(x^2-4x)^2} \end{aligned}$$

which need not be further simplified.

5. **(4 points)** Evaluate $\frac{d}{dt} [(t^3 - 3t + 2)(t + t^{-1})]$.

Using the product rule,

$$\begin{aligned} \frac{d}{dt} [(t^3 - 3t + 2)(t + t^{-1})] &= \left[\frac{d}{dt} (t^3 - 3t + 2) \right] (t + t^{-1}) + (t^3 - 3t + 2) \frac{d}{dt} (t + t^{-1}) \\ &= (3t^2 - 3)(t + t^{-1}) + (t^3 - 3t + 2)(1 - t^{-2}) \end{aligned}$$

which need not be further simplified.

6. **(2 point bonus)** Using only the product rule and the fact that $\frac{d}{dx} x = 1$, determine $\frac{d}{dx} \sqrt{x}$.

If we only knew the derivative of x , it would behoove us to relate \sqrt{x} to x in some way. The obvious way to do so is to note that $\sqrt{x} \cdot \sqrt{x} = x$. Then, taking the derivative of each side of this equation, we would get

$$\frac{d}{dx} (\sqrt{x} \cdot \sqrt{x}) = \frac{d}{dx} x$$

The right side we have an evaluation for by the information we are given, but the left side could only be expanded by the product rule:

$$\left(\frac{d}{dx} \sqrt{x} \right) \sqrt{x} + \sqrt{x} \left(\frac{d}{dx} \sqrt{x} \right) = 1$$

and then we may algebraically simplify to isolate the derivative we seek:

$$2\sqrt{x} \frac{d}{dx} \sqrt{x} = 1 \implies \frac{d}{dx} \sqrt{x} = \frac{1}{2\sqrt{x}}$$