1. **(15 points)** *Answer the following questions about the function* $f(x) = x^3 + 3x^2 - 24x$.

   (a) **(5 points)** *For which values of* x *is this function increasing? For which values is it decreasing? Label which is which.*

   The increase or decrease of $f(x)$ depends exclusively on the sign of $f'(x)$. Using the power rule, it can be easily determined that $f'(x) = 3x^2+6x-24 = 3(x^2+2x-8) = 3(x+4)(x-2)$ (the factorization step can be plausibly replaced with an invocation of square-completion or the quadratic formula).

   If $x < -4$, then both $x+4$ and $x-2$ are negative, so $f'(x)$ would be the product of two negative numbers and thus positive. If $-4 < x < 2$, then $x+4$ is positive but $x-2$ is negative, so $f'(x)$ is negative, as the product of a positive and negative number. Finally, if $x > 2$, both $x+4$ and $x-2$ are positive, so their product is as well. Thus, $f(x)$ is increasing when $f'(x)$ is positive, which is when $x < -4$ or $x > 2$; $f(x)$ is decreasing when $f'(x)$ is negative, which is when $-4 < x < 2$.

   (b) **(5 points)** *Where are this function's extrema? Identify each as a local maximum or a local minimum.*

   We see two sign transitions in $f'(x)$: at $x = -4$, $f(x)$ changes from increasing to decreasing, so $x = -4$ is a local maximum. at $x = 2$, $f(x)$ changes from decreasing to increasing, so $x = 2$ is a local minimum. These results could also be found via the second derivative test.

   (c) **(5 points)** *Where is the function concave up? Where is it concave down? Where are its points of inflection, if any? Label which is which.*

   The concavity of a function depends on its second derivative. The power rule will easily yield $f''(x) = 6x + 6$, so $f(x)$ is concave up when $f''(x) > 0$, which can algebraically be shown to be the case when $x > -1$; likewise, it is concave down when $f''(x) < 0$, which is the case when $x < -1$. The transition between these two behaviors is the point of inflection, at $x = -1$.

2. **(5 points)** *Determine where the function* $g(x) = \frac{x^2+3}{x+1}$ *is increasing and where it is decreasing. Label which is which.*

   We calculate the derivative with the quotient rule:

$$g'(x) = \frac{(x+1)(2x) - (x^2+3)(1)}{(x+1)^2} = \frac{x^2+2x-3}{(x+1)^2} = \frac{(x-1)(x+3)}{(x+1)^2}$$

   The factorization at the end could be effected with the quadratic formula or with square completion, if desired. The sign of this expression is dictated by the sign of its several factors: the division by $(x+1)^2$ will have no effect, since $(x+1)^2$ is nonnegative everywhere, but the $(x-1)$ and $(x+3)$ in the numerator may be negative or positive.

   If $x < -3$, then both $x-1$ and $x+3$ are negative, so $g'(x)$ would be the product of two negative numbers divided by a positive number and thus positive. If $-3 < x < 1$, then $x-1$ is negative but $x+3$ is positive, so $g'(x)$ is negative, as the product of a positive and negative number divided by a positive number. Finally, if $x > 1$, both $x-1$ and $x+3$ are positive, so their product is as well. Thus, $g(x)$ is increasing when $g'(x)$ is positive, which is when $x < -3$ or $x > 1$; $g(x)$ is decreasing when $g'(x)$ is negative, which is when $-3 < x < 1$.