1. **(5 points)** *Find the maximum and minimum values of* $f(x) = x^3 - 3x + 2$ *on the interval* $[0, 2]$.

   $f'(x) = 3x^2 - 3$. This is defined everywhere, and is zero when $3x^2 - 3 = 0$, which simplifies to $x^2 = 1$, or $x = \pm 1$. Thus our candidates for maximum and minimum are those of these zeroes that lie within the interval $[0, 2]$ together with 0 and 2 themselves. Thus our candidates are 0, 1, and 2 (-1 is not in the interval $[0, 2]$). $f(0) = 2$, $f(1) = 0$, and $f(2) = 4$, so the function is maximized at $x = 2$, where $f(x) = 4$, and minimized at $x = 1$, where $f(x) = 0$.

2. **(7 points)** *We wish to enclose a rectangular garden with picket fences along the two sides, chainlink in the back, and no fence in the front. Chainlink fence costs $3 per foot; picket fencing costs $1. We have $60 to spend on fencing material. What is the area of the largest garden we can enclose?*

   Denote the width of our garden by x and its depth by y. Thus, our costs for fencing are y for each of the sides and $3x$ for the back fence, and our total cost is thus $3x + 2y$. Since our total cost must be \$60, it follows that $3x + 2y = 60$ is a constraint of this system in which our goal is to maximize the toal area xy. We rephrase our constraint as a relationship between x and y: $y = 30 - \frac{3}{2}x$ (or, equivalently, $x = 20 - \frac{3}{2}y$). We can substitute this relationship into the area formula to get area as a function solely of a single dimension:

   $$A(x) = x\left(30 - \frac{3}{2}x\right) = 30x - \frac{3}{2}x^2$$

   Let us note in passing the domain of x: $0 < x < 20$, indicating two trivial gardens, one of which has zero width, and one in which all of our funds are spent on chainlink fence, leaving no possible money to spend on the side pickets. Note each of these extreme cases is actually a terrible idea: $A(0) = A(20) = 0$, since they represent gardens of dimension 0×30 and 20×0 respectively.

   Our maximum must thus be achieved at an extremum of $A(x)$. $A'(x) = 30 - 3x$, which is defined everywhere, so extrema occur only when $30 - 3x = 0$, or $x = 10$. Our largest garden is thus given by the dimension $x = 10$, with area $A(10) = 30 \cdot 10 - \frac{3}{2} \cdot 10^2 = 150$ square feet (alternatively; $y = 30 - \frac{3}{2} \cdot 10 = 15$, so the garden is 10×15, with an area of 150).

3. **(4 points)** *Evaluate the expression* $27^{-1/2} \cdot 27^{1/6}$ *to get a rational answer.*

   Combining and then interpreting the exponent, we see that

   $$27^{-1/2} \cdot 27^{1/6} = 27^{-1/2+1/6} = 27^{-2/6} = 27^{-1/3} = \frac{1}{sqrt[3]27} = \frac{1}{3}$$

4. **(4 points)** *Evaluate the expression* $2\log_3 6 - \log_3 4 + \log_3 3$*, giving your result as an integer.*

   Using the laws of logarithms:

   $$2\log_3 6 - \log_3 4 + \log_3 3 = \log_3(6^2) - \log_3 4 + \log_3 3 = \log_3 \frac{6^2 \cdot 3}{4} = \log_3 27 = 3$$