1. Let G be a simple graph with n vertices and m edges. Show that G is a complete graph if and only if $m = \binom{n}{2}$.

2. Find graphs G and H such that H is a subgraph of G, but is not an induced subgraph of G.

3. If a graph G has n vertices and m edges, is there a maximum length for a walk, path, or trail? If these maximums exist, what are they and why? If they do not exist, why not?

4. If a graph G has n vertices and m edges, prove that the minimum vertex degree δ and maximum vertex degree Δ in the graph are subject to the equation $\delta \leq \frac{2m}{n} \leq \Delta$.

5. Either give an example of a 3-regular graph on 7 vertices, or prove it cannot exist.

6. **Bonus:** Prove that a simple bipartite graph on n vertices can have no more than $\frac{n^2}{4}$ edges.