Due on Wednesday, January 28.

1. Let G_K be a graph with 64 vertices, one for each square of an 8×8 chessboard, and let two vertices be adjacent if a chess king (capable of moving a single square orthogonally or diagonally) could move from one to the other. Let G_Q, G_R, G_B, and G_N be similarly constructed, but with adjecency determined by queen's moves (movement any distance orthogonally or diagonally), rook's moves (movement any distance orthogonally only), bishop's moves (movement any distance diagonally only), or knight's moves (movement 2 squares in one direction and one square perpendicularly).

   (a) Which of these graphs are connected? For those of these graphs that are not connected, how many connected components do they have?

   (b) What is the distance between the two furthest apart vertices in a single component of G_K, G_Q, G_R, or G_B? (hint: what does distance in this graph measure with respect to the game itself?)

   (c) **Bonus:** what is the distance between the two furthest apart vertices in G_N?

2. The *diameter* of a graph G, sometimes denoted $d(G)$, is the distance between the two furthest apart vertices in the graph, i.e.

$$d(G) = \max_{u,v \in V(G)} d_G(u, v)$$

   The *radius* of a graph, denoted $r(G)$ is the minimum distance from a vertex v to the furthest vertex away from it, i.e.

$$r(G) = \min_{v \in V(G)} \max_{u \in V(G)} d_G(u, v)$$

   (the vertex v which has minimum distance from its furthest vertex is sometimes called a *center* of the graph, keeping with the circle metaphor).

   (a) Prove that $r(G) \leq d(G) \leq 2r(G)$.

   (b) Find a nontrivial graph in which $r(G) = d(G)$.

   (c) Find a nontrivial graph in which $d(G) = 2r(G)$.

3. **Bonus:** Suppose that a graph G contains a cycle C, and that there is a path in G of length k between some two vertices of C . Show that G contains a cycle of length at least $\sqrt{k}$.