Due on Wednesday, February 18.

1. **(10 points)** Let $B_1$ and $B_2$ be two blocks of a connected simple graph $G$. Prove that if $B_1$ and $B_2$ have a common vertex $v$, the graph $G - v$ produced by removing $v$ from $G$ is disconnected.

2. **(20 points)** Given a connected simple graph $G$, let a graph $H$ be produced in the following manner: let the blocks $B_1$, $B_2$, …, $B_k$ of the graph $G$ be associated with vertices $v_1$, $v_2$, …, $v_k$ of $H$, and let $v_i$ and $v_j$ be adjacent if and only if $B_i$ and $B_j$ have a common vertex. Prove the following two facts:

   (a) **(10 points)** $H$ is connected.

   (b) **(10 points)** $H$ is acyclic (the result from problem #1 may be useful here, and may be assumed to be true).

3. **(10 points)** We showed in class that the connectivity $\kappa(G)$ of a graph was no more than its minimum degree $\delta(G)$ — but in fact it can be far smaller! Demonstrate that a simple graph can have arbitrarily high minimum degree and still have very small connectivity.

4. **(10 points)** Let a connected (not necessarily simple) graph $G$ have subgraphs $G_1$, $G_2$, …, $G_k$ such that each edge of $G$ lies in exactly one $G_i$ (that is, the graphs $G_1, \ldots, G_k$ partition the edges of $G$). Prove that if there is an Eulerian circuit on each of the $G_i$, there is an Eulerian circuit on $G$. Is the converse true?

5. **(5 point bonus)** The *complement* $G^c$ of a graph $G$ is a graph on the same vertex set such that vertices $u$ and $v$ are adjacent in $G^c$ if and only if they are non-adjacent in $G$. Prove that either $G$ or $G^c$ is connected.