Due on Wednesday, March 11.

1. **(10 points)** Recall that G^c is the graph in which all non-adjacent vertices of G are adjacent and vice versa. Prove that if G has 11 or more vertices, G and G^c cannot both be planar.

2. **(5 points)** Prove that for all graphs G, $\alpha(G) + \omega(G) \leq n + 1$.

3. **(35 points)** The following is a somewhat simpler construction than that shown in class for constructing graphs with low clique number and high coloring number: for a graph G with vertices $v_1, \ldots, v_n$, let $M(G)$ be a graph with vertex-set $x_1, \ldots, x_n, y_1, \ldots, y_n, z$ (for $2n+1$ vertices total). For every edge $v_i v_j$ in G, include the following edges in $M(G)$: $x_i x_j$, $x_i y_j$, $x_j y_i$. In addition, include the edges $y_1 z$, $y_2 z$, $y_3 z$, $\ldots$, $y_n z$.

   (a) **(5 points)** Show that for any graph G, $\chi(M(G)) \leq \chi(G) + 1$.

   (b) **(10 points)** Show that $\chi(M(G)) \geq \chi(G) + 1$ (hint: it may be easier to start by showing that $\chi(M(G)) \geq \chi(G)$).

   (c) **(10 points)** Show that if G does not have a K_3 subgraph, neither does $M(G)$.

   (d) **(5 points)** Let $M_2 = K_2$ and $M_i = G(M_{i-1})$ for $i > 2$. Prove using the results from the above sections that $\chi(M_i) = i$ and $\omega(M_i) = 2$.

   (e) **(5 points)** Prove that M_i has $3 \cdot 2^{i-2} - 1$ vertices.

4. **(5 point bonus)** If G is a planar graph with n vertices without any C_3 subgraphs, what is the maximum number of edges that G can have? What if G lacks both C_3 and C_4 subgraphs?