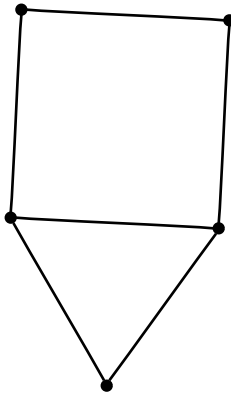


1. (10 points) Describe (either in words or a diagram) a  $K_6$ -free graph on 11 vertices with as many edges as possible. How many edges does it have?

2. (10 points) Find at three distinct acyclic orientations of the following graph.



3.

4. **(40 points)** For each of the following statements, either prove it (if true) or give a counterexample (if false).

(a) There is no digraph  $D$  such that  $d^+(u) \leq d^-(u)$  for all vertices of  $D$ .

(b) There is a value of  $n$  such that the random graph  $G(n, \frac{1}{2})$  has a probability of at least 75% of containing  $C_6$  as an induced subgraph.

(c) If  $D$  is an acyclic orientation of a graph  $G$  and  $G$  has a Hamiltonian cycle, then  $D$  has a directed Hamiltonian path.

(d) If  $\|G\| \geq \frac{|G|}{2}$ , then  $G$  has  $P_3$  as a subgraph.