

1. **(4 points)** Solve the following system of equations. Show your work.

$$\begin{cases} 3x - 2y = 1 \\ x + 3y = -7 \end{cases}$$

The cleanest substitution uses the second equation to express  $x$  in terms of  $y$ :  $x = -7 - 3y$ . Substituting this back into the first line eliminates  $x$ , leaving an equation in  $y$ :

$$\begin{aligned} 3(-7 - 3y) - 2y &= 1 \\ -21 - 9y - 2y &= 1 \\ -11y &= 22 \\ y &= -2 \end{aligned}$$

And then by our original solution for  $x$  in terms of  $y$ ,  $x = -7 - 3(-2) = -7 + 6 = -1$ , so the solution is  $x = -1$ ,  $y = -2$ , or  $(-1, -2)$ .

2. **(6 points)** In pre-decimal British currency, a sixpence is a coin worth 6 cents, and a penny is a coin worth 1 cent. We have a collection of 70 coins of these two types which have a total value of 240 cents (or a pound sterling). Set up and solve a system of equations to determine how many coins of each type we have.

Let  $s$  be the number of sixpence in our collection; let  $p$  be the number of pennies. Since there are 70 coins in total,  $s + p = 70$ , and since the total value of the coins is 240 cents,  $6s + p = 240$ , so our system is

$$\begin{cases} s + p = 70 \\ 6s + p = 240 \end{cases}$$

Using the substitution method, we can solve for  $s$  in terms of  $p$  in the first equation:  $s = 70 - p$ , and then substitute back in to the second equation:

$$\begin{aligned} 6(70 - p) + p &= 240 \\ 420 - 6p + p &= 240 \\ -5p &= -180 \\ p &= 36 \end{aligned}$$

and from our original solution for  $s$ , we know  $s = 70 - p = 34$ , so this collection includes 36 pennies and 34 sixpence.

3. **(5 points)** Solve the following system of equations. Show your work.

$$\begin{cases} x - 2y + z = -5 \\ 2x + 3y = 8 \\ 3x - 6y - 4z = -1 \end{cases}$$

Using the substitution method, we can solve for  $x$  to get  $x = -5 + 2y - z$ , which substitutes into the other two equations to give:

$$\begin{cases} 2(-5 + 2y - z) + 3y = 8 \\ 3(-5 + 2y - z) - 6y - 4z = -1 \end{cases}$$

We now distribute the multiplication and collect terms to get

$$\begin{cases} 7y - 2z = 18 \\ -7z = 14 \end{cases}$$

The last equation gives  $z = -2$ ; substituting into its predecessor, we have  $7y = 18 + 2z = 18 - 4 = 14$ , so  $y = 2$ ; finally, since  $x = -5 + 2y - z$ , it follows that  $x = -5 + 2(2) - (-2) = 1$ .