1. **(4 points)** *Identify each of the following sequences as arithmetic, geometric, neither, or both. If a sequence is arithmetic or geometric, determine its common difference or ratio:*

   - $6, 0, -6, 0, 6, 0, -6, \ldots$.
     This sequence cannot possibly be geometric, as the first ratio doesn't even exist (since 6 cannot be divided by 0)! It is likewise not arithmetic; the first difference is $0-6=-6$, and the second is $-6-0=-6$, which is encouraging, but the third difference is $0-(-6)=6$, which is unequal to the first two differences, so there is no common difference. Thus this sequence is neither arithmetic nor geometric.
   - $2, -4, 8, -16, 32, -64, \ldots$.
     This sequence is not arithmetic, since the first difference of successive terms is $-4-2=-6$ while the second is $8-(-4)=12$. However, it is geometric, as all of the ratios are identical:
     $$\frac{-4}{2} = \frac{8}{-4} = \frac{-16}{8} = \frac{32}{-16} = \frac{-64}{32} = -2$$
     so this is a geometric sequence of common ratio $-2$.
   - $5, \frac{3}{2}, -2, \frac{-11}{2}, -9, \ldots$.
     This sequence is arithmetic, since all the differences of successive terms are equal:
     $$\frac{3}{2} - 5 = -2 - \frac{3}{2} = \frac{-11}{2} + 2 = -9 - \frac{-11}{2} = \frac{-7}{2}$$
     so it is an arithmetic sequence of common difference $\frac{-7}{2}$. It is verifiable non-geometric for any of a number of reasons: for one, because it is arithmetic and nonconstant; or, alternatively, by inspecting the first two ratios: $\frac{5}{3/2} = \frac{10}{3}$, which is not equal to $\frac{3/2}{-2} = \frac{-3}{4}$.
   - $25, 15, 9, \frac{27}{5}, \frac{81}{25}, \ldots$.
     This sequence is demonstrably non-arithmetic: $15-25=-10$, while $9-15=-6$, so there is no common difference. However, it will be geometric, as all of the ratios of successive terms are equal:
     $$\frac{15}{25} = \frac{9}{15} = \frac{27/5}{9} = \frac{81/25}{27/5} = \frac{3}{5}$$
     so it is a geometric sequence of common ratio $\frac{3}{5}$.

   **Grading: one point for fully correct answer (both type of sequence and, for the last 3 questions, the common difference/ratio). Half a point on the last three for the correct type without common difference/ratio.**

2. **(4 points)** *An arithmetic sequence has fifth term* 16 *and eighth term* 7*. What is its second term?*

   Since the sequence is arithmetic, we know it conforms to the template $a_n = a_1 + (n-1)d$, and we are given the two specific terms $a_5 = 16$ and $a_8 = 7$. Plugging these specific values into the template, we get
   $$\begin{aligned} 16 &= a_5 = a_1 + 4d \\ 7 &= a_8 = a_1 + 7d \end{aligned}$$
   These two equations, collectively, are a system of two linear equations in the two unknowns $a_1$ and $n$, so we solve for them by either elimination or substitution (elimination is used below).

If we subtract the first equation from the second, the $a_1$ terms will be eliminated:

$$7 - 16 = (a_1 + 7d) - (a_1 + 4d)$$

to leave us with the simple equation $-9 = 3d$, so $d = -3$.

We can acquire $a_1$ by substituting our known equation into either of the two original equations in our system, so for instance, if we use the first, we have $16 = a_1 = 4(-3)$, so $a_1 = 16+12 = 28$. The formula for the $n$th term of our sequence is thus $a_n = 28 + (n-1)(-3)$. In particular, the second term is $a_2 = 28 + (1)(-3) = 25$.

**Grading: One point for crafting a system of two equations, one point for determining each of $a_1$ and $d$, and one point for finding $a_2$. Deduct no more than one point for arithmetic errors as long as the process is correct. If the student uses a method which does not involve calculating $a_1$ directly, subsume those points into the calculation of $a_2$.**

3. **(4 points)** *Find the sum of the finite arithmetic series* $6 + 1 - 4 - 9 - 14 - 19 - \cdots - 94$.

The underlying arithmetic sequence obviously has $a_1 = 6$, and slightly less obviously but straightforwardly has $d = -5$. The formula for the $n$th term is thus $a_n = 6 - 5(n-1)$. We wish to determine how many terms this series has; that is, which term of the sequence is equal to $-94$? We substitute $-94$ in as the result of our formula and solve for $n$:

$$\begin{aligned} -94 &= 6 - 5(n-1) \\ -100 &= -5(n-1) \\ 20 &= n - 1 \\ 21 &= n \end{aligned}$$

so this series has 21 terms. We may now use the "Gauss" trick of adding the series to itself:

$$\begin{aligned} 2S = (6-94) + (1-89) + (-4-84) + \cdots + (-94+6) \\ 2S = -88 - 88 - 88 - \cdots - 88 \\ S = -44 - 44 - 44 - \cdot - 44 = -44 \cdot 21 = -924 \end{aligned}$$

or, alternatively, we can use the formula:

$$\begin{aligned} S &= na_1 + \frac{n(n-1)}{2}d \\ &= 21 \cdot 6 + \frac{21 \cdot 20}{2}(-5) \\ &= 126 - 1050 = -924 \end{aligned}$$

**Grading: Two points for determining that there are 21 terms in the series, two points for actually adding them up; deduct one point for arithmetic errors, and only 2 if there is a profound error in an otherwise decent process.**

4. **(3 points)** *Find the sum of the infinite geometric series* $5 - 3 + \frac{9}{5} - \frac{27}{25} + \frac{81}{125} - \cdots$

This series has common ratio $\frac{-3}{5}$ and first term 5. Since $|\frac{-3}{5}| < 1$, the infinite sum exists, and its actual value is $\frac{a_1}{1-d} = \frac{5}{1+3/5} = \frac{5}{8/5} = \frac{25}{8}$.

**Grading: One point for identifying $d$, one for identifying $a_1$, one for invoking the formula.**