1. **(3 points)** *Solve the equation* $3y^2 + 2y = 1 - 4y$.

First, we subtract $1 - 4y$ from both sides to get an equality with a quadratic on one side and zero on the other; then we have $3y^2 + 6y - 1 = 0$. Now there are two plausible ways to continue: square-completion, or the quadratic formula. Factorization is not a recommended method, since this quadratic will not factor easily.

To use the quadratic formula, we plug in the values $a = 3$, $b = 6$, and $c = -1$ to get:

$$y = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-6 \pm \sqrt{6^2 - 4(3)(-1)}}{2 \cdot 3} = \frac{-6 \pm \sqrt{36 + 12}}{6} = -1 \pm \frac{\sqrt{48}}{6}$$

This can be simplified slightly further, since $\sqrt{48} = \sqrt{16 \cdot 3} = 4\sqrt{3}$, so $y = -1 \pm \frac{2\sqrt{3}}{3}$.

Alternatively, one could use square-completion; we might simplify matters by first dividing by the coefficient of 3 on y^3 to get the equation $y^2 + 2y - \frac{1}{3} = 0$. The term that completes a square with $y^2 + 2y$ is 1, so we shall add $\frac{4}{3}$ to both sides of the equation to get:

$$\begin{aligned} y^2 + 2y + 1 &= \frac{4}{3} \\ (y+1)^2 &= \frac{4}{3} \\ y + 1 &= \pm\sqrt{\frac{4}{3}} \\ y = -1 \pm \sqrt{\frac{4}{3}} &= -1 \pm \frac{2}{\sqrt{3}} \end{aligned}$$

which, after rationalizing the denominator, is again $y = -1 \pm \frac{2\sqrt{3}}{3}$.

2. **(5 points)** *Solve the equation* $x + \sqrt{x-2} = 2$.

We have a radical in this equation, so it is best solved by isolating the radical term and then squaring both sides. We algebraically isolate $\sqrt{x-2}$ by subtracting 2 from both sides, and then we square as suggested:

$$\begin{aligned} \sqrt{x-2} &= 2 - x \\ \left(\sqrt{x-2}\right)^2 &= (2-x)^2 \\ x - 2 &= 4 - 4x + x^2 \\ 0 &= x^2 - 5x + 6 \end{aligned}$$

so we have reduced this radical equation to the quadratic equation $x^2 - 5x + 6 = 0$. This can be solved using the quadratic formula, completion of the square, or factorization; the results of invoking the quadratic formula are shown here:

$$x = \frac{5 \pm \sqrt{5^2 - 4 \cdot 6 \cdot 1}}{2} = \frac{5 \pm 1}{6} = 3 \text{ or } 2$$

so our solution process has yielded $x = 2$ and $x = 3$ as *potential* solutions; however, since we squared a radical in the process of solving it, we may have introduced extraneous solutions, so we need to check both of these agains the original equation to see if they solve it:

$$\text{When } x = 2\text{: } x + \sqrt{x-2} = 2 + \sqrt{2-2} = 2 + 0 = 2$$
$$\text{When } x = 3\text{: } x + \sqrt{x-2} = 3 + \sqrt{3-2} = 3 + 1 \neq 2$$

So $x = 3$ is *not* a solution of the original equation; only $x = 2$ is.

3. **(3 points)** *Solve the inequality* $1 - \frac{t}{3} < 2$. *Write the solution in either interval notation or as conditions on* t.

   We solve this inequality like an equality, being careful to invert the direction of the inequality when we multiply or divide by a negative number:

$$\begin{aligned} 1 - \frac{t}{3} &< 2 \\ -1 \qquad & \quad -1 \\ -\frac{t}{3} &< 1 \\ t &> -3 \end{aligned}$$

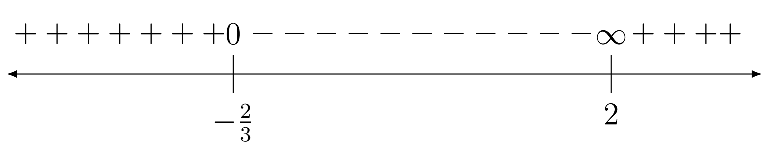
   This can also be written in interval notation as $(-3, \infty)$.

4. **(4 points)** *Solve the inequality* $\frac{3x+2}{x-2} \leq 0$. *Write the solution in either interval notation or as conditions on* x.

   Here we want to know where $\frac{3x+2}{x-2}$ is nonnegative. We start by identifying the points where it is zero or nonexistent. It is nonexistent when its denominator is zero; this occurs when $x - 2 = 0$, or in other words, at $x = 2$. It is zero when its numerator is zero; this occurs when $3x + 2 = 0$, or in other words at $x = \frac{-2}{3}$. We thus have, so far, a sketch of the value of the expression $\frac{3x+2}{x-2}$ at two significant points:

   Note that ∞ is here used as a shorthand for "does not exist". Other notations like "DNE" or "X" will also work, depending on your preference.

   Now, in the three regions into which these two significant points divide the number line, we must probe the values of $\frac{3x+2}{x-2}$. From the leftmost region, a good representative point would be testing $x = -1$, which gives $\frac{3x+2}{x-2} = \frac{3(-1)+2}{-1-2} = \frac{-1}{-3}$, which is a positive number, so on the leftmost region of the number line, $\frac{3x+2}{x-2}$ is positive. Likewise, we can test the middle region with $x = 0$ to get $\frac{3x+2}{x-2} = \frac{0+2}{0-2} = -1$, a negative number, and we can test the rightmost region with $x = 3$ to get $\frac{3x+2}{x-2} = \frac{3 \cdot 3+2}{3-2} = 11$, a positive number, so our eventual sign-investigation plotted on a number line looks like:



so the values where $\frac{3x+2}{x-2} \leq 0$ correspond to the points on this number line marked with a "0" or "−". These points are $-\frac{2}{3}$ and every point between $-\frac{2}{3}$ and 2; we can write this as a condition on x as $-\frac{2}{3} \leq x < 2$, or as an interval $[-\frac{2}{3}, 2)$.