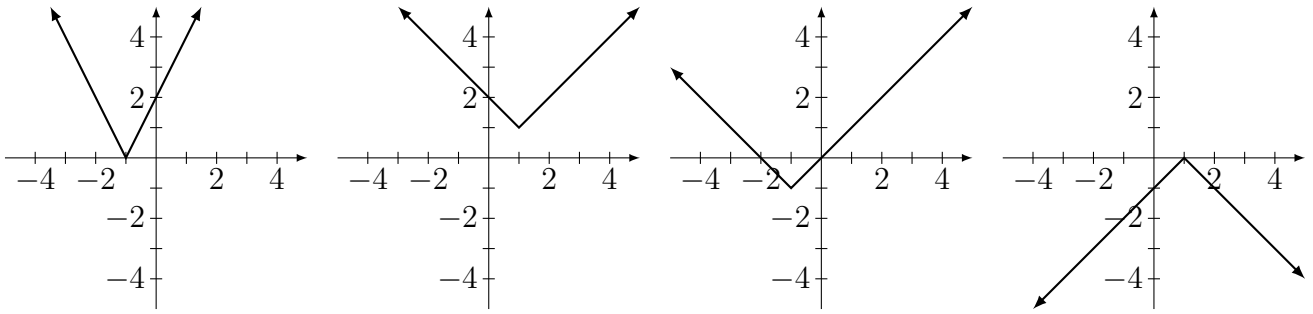


1. (4 points) The following four graphs are of the equations $y = -|x - 1|$, $y = |x + 1| - 1$, $y = |x - 1| + 1$, and $2|x + 1|$. Label which is which:



Two of the graphs are easily identified; the first has sections which have been stretched vertically and the last has been flipped vertically from the usual shape of $y = |x|$. Thus the first graph corresponds to $y = 2|x + 1|$ (which has a stretch evidenced by the outer multiplication by 2), and the fourth to $-|x - 1|$ (which has a vertical flip evidenced by the outer multiplication by -1).

The remaining two are subtler. $y = |x + 1| - 1$ consists of a subtraction of 1 from the absolute value (which corresponds to a shift of 1 unit downwards), and an addition of 1 to x itself (which corresponds to a shift of 1 unit leftwards). Thus we want an absolute value curve shifted left and downwards, which describes the third graph above. Finally, $y = |x - 1| + 1$ corresponds to the second graph, either by process of elimination or by observing that this function corresponds to an upwards and rightwards shift of the graph of $y = |x|$.

2. (4 points) Describe in words, using specific quantities and directions when necessary, the transformation to the graph of $f(x) = \sqrt{x}$ needed to get the graphs of the following functions:

- $g(x) = \sqrt{x - 3}$.

Here $g(x) = f(x - 3)$, which is a shift of the graph of $f(x)$ by 3 units to the right.

- $h(x) = \sqrt{-x}$.

Here $h(x) = f(-x)$, which is a horizontal flip of $f(x)$.

- $r(x) = \sqrt{5x}$.

Here $r(x) = f(5x)$, which is a squashing of $f(x)$ by a factor of 5 horizontally.

- $q(x) = \sqrt{x} + 2$.

Here $q(x) = f(x) + 2$, which is a shift of the graph of $f(x)$ by 2 units upwards.

3. (4 points) Let $f(x) = \frac{1}{x+2}$ and $g(x) = \frac{x^2}{x-1}$. Write out explicit formulas for the following functions. Do not algebraically simplify your results.

- (3 points) $(f \circ g)(x)$.

$$f \circ g(x) = f(g(x)) = f\left(\frac{x^2}{x-1}\right) = \frac{1}{\frac{x^2}{x-1} + 2}.$$

- (1 point) $(g - f)(x)$.

$$(g - f)(x) = g(x) - f(x) = \frac{x^2}{x-1} - \frac{1}{x+2}.$$

4. (4 points) As above, let $f(x) = \frac{1}{x+2}$ and $g(x) = \frac{x^2}{x-1}$. Determine the domains of $(f + g)(x)$, $(f - g)(x)$, $(fg)(x)$, and $\frac{f}{g}(x)$, expressed either in interval notation or as conditions on x . Label which is which.

We can note easily that the domain of $f(x)$ excludes the value -2 , since $\frac{1}{x+2}$ has a denominator that cannot be zero; likewise $g(x)$ has a domain excluding 1 .

The domains of $(f + g)(x)$, $(f - g)(x)$, and $(fg)(x)$ are all merely the overlap of the domains of $f(x)$ and $g(x)$; that is to say, the domain consists of values of x which are neither -2 nor 1 ; as conditions on x , we might write this as $x \neq -2, 1$; as intervals we might express it as $(-\infty, -2) \cup (-2, 1) \cup (1, \infty)$.

The domain of $\frac{f}{g}(x)$ contains all elements of the above-determined domain where $g(x) \neq 0$; Since $g(x) = \frac{x^2}{x-1}$, it is easy to see that $g(x) = 0$ when $x = 0$; thus we must exclude 0 from the domain; since we already excluded -2 and 1 , we shall see that $\frac{f}{g}(x)$ has domain given by $x \neq -2, 0, 1$, or, in interval form, as $(-\infty, -2) \cup (-2, 0) \cup (0, 1) \cup (1, \infty)$.