1. **(4 points)** *Either demonstrate that the function $f(x) = 5 - 2x^3$ has no inverse function, or find a formula for its inverse.*

   We can solve for the inverse by noting that it must satisfy $f(f^{-1}(y)) = y$, so:

$$\begin{aligned} 5 - 2[f^{-1}(y)]^3 &= y \\ -2[f^{-1}(y)]^3 &= y - 5 \\ [f^{-1}(y)]^3 &= \frac{y-5}{-2} \\ f^{-1}(y) &= \sqrt[3]{\frac{y-5}{-2}} \end{aligned}$$

2. **(3 points)** *Find the quadratic function $y = f(x)$ which has vertex $(-3, 4)$ and which passes through the point $(0, 0)$.*

   The standard form $y = a(x-h)^2 + k$ describes a quadratic with vertex at (h, k), so in this specific case we would put in $h = -3$ and $k = 4$ to find that our quadratic ought to have the form $y = a(x+3)^2 + 4$. But we still need to solve for a. Since the quadratic passes through $(0, 0)$, the equation must be satisfied when $x = 0$ and $y = 0$:

$$\begin{aligned} 0 &= a(0+3)^2 + 4 \\ -4 &= a \cdot 3^2 \\ \frac{-4}{9} &= a \end{aligned}$$

   so our formula will be $y = \frac{-4}{9}(x+3)^2 + 4$

3. **(4 points)** *You are planting cherry tomatoes in a small garden; overcrowding will reduce each plant's production. If you put only 4 plants in the garden, they will each produce 80 fruits over the course of the season. Each additional plant will reduce the per-plant productivity by 5 fruits. How many cherry tomatoes should you plant to maximize your yield? Show your work.*

   Let x be the number of plants put in the garden. When $x = 4$, each plant yields 80 tomatoes, but for each plant beyond four, we lose 5 fruits per plant. The number of plants by which we are exceeding 4 is $(x - 4)$, so the lost productivity after putting x plants down would be $5(x-4)$; thus the per-plant yield after planting x plants is $80 - 5(x-4)$. Since we have x plants in total, our total productivity would be the product of the number of plants and the number of fruits per plant, which would be $x \cdot [80 - 5(x-4)] = 80x - 5x^2 + 20x = -5x^2 + 100x$. Since we wish to maximize our yield, we want to know which value of x maximizes $-5x^2 + 100x$. THis is a quadratic, whose vertex always occurs at $x = \frac{-b}{2a}$, so in this case the maximum occurs at $x = \frac{-100}{2(-5)} = 10$, so 10 plants is an optimal quantity.

4. **(4 points)** *Determine the end behavior (a.k.a. long-term behavior), x-intercepts (a.k.a. zeroes), and y-intercepts of the function $g(x) = (-3x+2)(x-4)^2$.*

   This function would, when multiplied out, have leading term $-3x^3$; since this has an odd exponent and negative coefficient, the arrows would be in the upper left and lower right of the plane; alternatively, you could say that $g(x)$ applied at large values swaps the signs of its input (large negatives are mapped to large positives, and vice versa). The y-intercept is $g(0) = (-3 \cdot 0 + 2)(0-4)^2 = 2(-4)^2 = 32$. The x-intercepts are those points where $g(x) = 0$;

that is, the solutions to the equation $(-3x+2)(x-4)^2 = 0$. The product on the left side equals zero when either of the multiplicands equals zero, so either $-3x+2=0$ or $x-4=0$; these occur respectively when $x = \frac{2}{3}$ or $x = 4$. Thus the x-intercepts are $\frac{2}{3}$ and 4.