1. **(3 points)** *Write the expression* $\frac{1}{5}(\ln z - 2\ln y)$ *in condensed form (i.e. as a single logarithm).*

   Using in turn the exponent rule $m\log_b x = \log_b(x^m)$, the quotient rule $\log_b x - \log_b y = \log_b \frac{x}{y}$, and the exponent rule again:

$$\frac{1}{5}(\ln z - 2\ln y) = \frac{1}{5}(\ln z - \ln(y^2)) = \frac{1}{5}\left(\ln\frac{z}{y^2}\right) = \ln\left(\frac{z}{y^2}\right)^{1/5}$$

   Variant forms of this final answer are possible, such as $\ln\sqrt[5]{\frac{z}{y^2}}$ or $\ln(z^{1/5}y^{-2/5})$.

2. **(4 points)** *Solve the equation* $5^{2x+1} = 1$.

   We take the base 5 logarithm of both sides to clear out the exponentiation:

$$\begin{aligned} 5^{2x+1} &= 1 \\ \log_5(5^{2x+1}) &= \log_5 1 \\ 2x+1 &= 0 \\ 2x &= -1 \\ x &= \frac{-1}{2} \end{aligned}$$

3. **(5 points)** *Solve the equation* $\log_3(2x+3) - \log_3(x-5) = 1$.

   We use the quotient rule to consolidate the logarithms, and then use both sides as the exponent on a base of 3 to cancel the logarithms.

$$\begin{aligned} \log_3(2x+3) - \log_3(x-5) &= 1 \\ \log_3\frac{2x+3}{x-5} &= 1 \\ 3^{\log_3\frac{2x+3}{x-5}} &= 3^1 \\ \frac{2x+3}{x-5} &= 3 \\ 2x+3 &= 3(x-5) \\ 2x+3 &= 3x-15 \\ 18 &= x \end{aligned}$$

   so $x = 18$ is our solution.

4. **(3 points)** *Calculate the values of the following three logarithms; write your final answer as a number without logarithms:*

   - $\log_{16} 2$.
     We know that $2^4 = 16$, but that's not a relationship describing 2 as 16 raised to a power. We can rephrase this relationship, however, as $2 = \sqrt[4]{16} = 16^{1/4}$, so $\log_{16} 2 = \frac{1}{4}$.
   - $\log_3 \frac{1}{27}$.
     We know that $3^3 = 27$, so $\frac{1}{27} = \frac{1}{3^3} = 3^{-3}$. Thus, $\log_3\frac{1}{27} = -3$.
   - $\log_7 49$.
     We know that $7^2 = 49$, so $\log_7 49 = 2$.