This test is closed-book and closed-notes. No calculator is allowed for this test. For full credit show all of your work (legibly!), unless otherwise specified. Radical, exponential, trigonometric, and inverse trigonometric expressions may be left unsimplified.

1. **(20 points)** Consider the vector-valued function $\mathbf{r}(t) = \left\langle \frac{t^3}{3}, -2t, t^2 \right\rangle$

   (a) **(5 points)** Find the parametric equations of a tangent line to the curve described by $\mathbf{r}(t)$ at the point $(9, -6, 9)$.

   (b) **(5 points)** Find the arclength along this curve from $(0, 0, 0)$ to $(9, -6, 9)$.

   (c) **(5 points)** Find the curvature of this curve at $(9, -6, 9)$.

   (d) **(5 points)** Find the unit binormal vector at $(9, -6, 9)$.

1	/20
2	/10
3	/15
4	/15
5	/(5)
Σ	/60

2. **(10 points)** Consider the line given by the system of parametric equations $x = 3t-5$, $y = -t+2$, $z = 4t + 4$, and the plane given by the equation $2x + 2y - z = 8$.

   (a) **(3 points)** Does the line intersect the plane or not?

   (b) **(7 points)** If the line intersects the plane, determine the point where it does so; if it does not intersect the plane, determine the distance between the line and plane.

3. **(15 points)** Answer the following questions about surfaces in space:

   (a) **(5 points)** Identify the surface described by the equation $z = x^2 + 3y^2 - 18y$ and state its orientation, if applicable.

   (b) **(5 points)** Determine (either by description or by sketching) the domain of the multivariable function $f(x, y) = \frac{\sqrt{1-x^2-y^2}}{x+y}$.

   (c) **(5 points)** Give a parametric system of equations describing the curve formed by the intersection of $z = x^2 + 3y^2 - 18y$ and $x + 2y = 4$.

4. **(15 points)** In the questions which follow, $\mathbf{u} = 2\mathbf{i} - 4\mathbf{j} + 6\mathbf{k}$ and $\mathbf{v} = \mathbf{i} - \mathbf{k}$.

   (a) **(4 points)** Identify each of the following four expressions as a vector, a scalar, or as uncalculatable nonsense. You do not need to calculate these expressions or justify your assertions!

   i. $(\mathbf{u} \cdot \mathbf{v}) - |\mathbf{v}|$.

   ii. $\frac{\mathbf{u}-\mathbf{v}}{\mathbf{v}}$.

   iii. $|\mathbf{u}|\mathbf{v} - (\mathbf{u} \times \mathbf{v})$.

   iv. $\frac{1}{|\mathbf{u}|}(\mathbf{u} \times \mathbf{v})$.

   (b) **(3 points)** Calculate $\mathbf{u} \times \mathbf{v}$.

   (c) **(4 points)** Find the angle between $\mathbf{u}$ and $\mathbf{v}$.

   (d) **(4 points)** Calculate $\text{proj}_{\mathbf{v}} \mathbf{u}$.

5. **(5 point bonus)** Prove on the back of this page that $\mathbf{a} \times (\mathbf{b} \times \mathbf{c}) = (\mathbf{a} \cdot \mathbf{c})\mathbf{b} - (\mathbf{a} \cdot \mathbf{b})\mathbf{c}$.