1. **(16 points)** Write but do not evaluate the following integrals:

   (a) **(6 points)** A cylindrical integral to calculate the volume of the solid which lies in the first octant (where x, y, and z coordinates are all positive) under the paraboloid $z = 2 - x^2 - y^2$ and above the cone $z = \sqrt{x^2 + y^2}$.

   (b) **(5 points)** A polar integral to calculate $\iint_D e^{-x^2-y^2} dA$, where D is the region given by $x^2 + y^2 \leq 4$ with $y \geq 0$ and $x \leq y$.

   (c) **(5 points)** A spherical integral to calculate $\iiint_E x^2 + y^2 dV$ where E is the hollow hemispherical shell given by $1 \leq x^2 + y^2 + z^2 \leq 9$ with $y \geq 0$.

1	/ 16
2	/ 16
3	/ 6
4	/ 8
5	/ 14
6	/ (5)
Σ	/ 60

2. **(16 points)** Calculate the following integrals, using whatever approach you find most effective:

   (a) **(6 points)** $\iiint_E 5y dV$ where E is the solid in the first octant bounded by the surfaces $x = 0$, $y = 0$, $y = x$, $z = x^2$, and $z = 4$.

   (b) **(6 points)** $\iint_D x - 2y dA$ where D is the region bounded by the curves $y = 2 - x^2$ and $y = x^2$.

   (c) **(4 points)** $\iint_D 2x + y dA$ where D is the rectangle with corners $(-3, 0)$, $(2, 0)$, $(2, 2)$, and $(-3, 2)$.

3. **(6 points)** Using the transformations $x = 2u - v$ and $y = u + 4v$, evaluate $\iint_D x + y dA$ over the region D bounded by $4x + y = 18$, $4x + y = 27$, $2y - x = 0$, and $2y - x = 9$.

4. **(8 points)** Determine whether each of the following vector fields as either conservative or nonconservative; for each that is conservative, find a potential function:

   - $F(x, y) = (4x - 3y + 2)\mathbf{i} + (4y + 1)\mathbf{j}$.

   - $G(x, y) = \langle \frac{2x}{y} + 3, y^2 - \frac{x^2}{y^2} \rangle$.

   - $H(x, y) = \langle \ln y - e^x, 7 \sin y + \frac{x}{y} \rangle$.

5. **(14 points)** Calculate the following path integrals

   (a) **(5 points)** $\int_C x^2 ds$ where C is the line segment from $(0, 4)$ to $(3, 2)$.

   (b) **(5 points)** $\int_C F \cdot d\mathbf{r}$, where $F(x, y, z) = \langle 4y + z, 3x - z, 2z \rangle$ and C is the curve given by $x = t$, $y = t^2$, and $z = t$ from $(0, 0, 0)$ to $(2, 4, 2)$.

   (c) **(4 points)** $\int_C 2x - 3y dy$ where C is the curve $y = x^2 - 1$ from $(1, 0)$ to $(4, 15)$.

6. **(5 points)** On the back of this sheet, identify the shape of the solid whose volume is described by the integral $\int_0^{2\pi} \int_{\pi/4}^{3\pi/4} \int_0^{3 \csc \phi} \rho^2 \sin \phi d\rho d\phi d\theta$, and calculate its volume *without taking an integral.*